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Some Considerations  
on the Steep-Dip Finite-Difference Migration

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## CONTENTS

|   | Page |
|---|------|
| Abstract.....                               | 1    |
| Introduction.....                           | 1    |
| Theory of finite-difference migration.....  | 1    |
| Migration test.....                         | 3    |
| Derivative-to-difference approximation..... | 8    |
| Conclusions.....                            | 13   |
| References cited.....                       | 13   |
| Appendix.....                               | 14   |

## ILLUSTRATIONS

|   |    |
|---|----|
| Figure 1. A W-shaped reflector model.....   | 4  |
| 2. Synthetic zero-offset section of fig. 1.....   | 5  |
| 3. Finite-difference migration of fig. 2 by an explicit<br>scheme (A), by an implicit second-order equation (B),<br>and by an implicit fourth-order equation (C)..... | 6  |
| 4. Finite-difference migration of fig. 2 by the implicit<br>fourth-order equation with reduced trace interval.....  | 7  |
| 5. Amplitude spectra of second-derivative operators.....  | 11 |

## TABLES

|   |    |
|---|----|
| Table 1. Run-time report on the finite-difference migrations.....             | 8  |
| 2. Estimated coefficients of the difference operators<br>in equation (9)..... | 10 |
| 3. Estimated time report on finite-difference migration<br>of fig. 2.....     | 12 |

# SOME CONSIDERATIONS ON THE STEEP-DIP FINITE-DIFFERENCE MIGRATION

By S. Y. Suh, M. W. Lee, and J. A. Grow

## ABSTRACT

The practical implementation of a steep-dip migration scheme by the finite-difference method is the main concern of this paper. Successful steep-dip migration requires not only an accurate one-way wave equation but also an accurate difference approximation of the differential equation.

Accurate one-way equations can be obtained using an optimization method (Lee and Suh, 1985). This optimization method can also be applicable to the derivation of accurate difference operators.

This paper describes a practical approach to the steep-dip migration using optimized one-way and difference operators.

## INTRODUCTION

Seismic migration is a process of imaging the subsurface from the measured data. The finite-difference method is one of the migration schemes, the advantage being that it is easily applicable to spatial velocity variations. However, this method cannot accurately image reflectors having high dip angles. The dip limitation of the finite-difference method is primarily due to the inaccurate dispersion relation of the approximate one-way equation employed in the scheme.

There are two types of one-way equations: explicit and implicit. Gazdag (1980) and Berkhout (1980) introduced a finite-difference method using explicit one-way equations. In this method, the derivatives in the horizontal direction ( $x$ ) are estimated either by a convolution in the spatial domain or by a multiplication in the wavenumber domain. This method effectively controls the difference-approximation error. Ma (1981) developed a solution method to high-order implicit equations. Lee and Suh (1985) introduced optimized one-way equations of the implicit type which significantly reduce the dispersion error in conventional one-way equations.

The steep-dip finite-difference migration by the implicit one-way equation is degraded by the error in derivative-to-difference approximation. The apparent wavenumber in  $x$ -direction increases from zero to the true wavenumber as the dip increases from zero to 90 degrees. Therefore, the difference error is more significant in the steep-dip migration than in the gentle-dip migration.

This paper is organized as follows. The first section reviews the theory of finite-difference migration. The second section describes a synthetic example on steep-dip finite-difference migration by the explicit and implicit schemes, which shows that each scheme has its own problem for the steep-dip migration. Finally, a new method of the difference approximation is presented. This paper also includes an appendix, which is a computer program written mainly in array-processor assembler language.

## THEORY OF FINITE-DIFFERENCE MIGRATION

The finite-difference migration employs an one-way equation for the wave-field extrapolation. According to the type of one-way equation, it is divided into two schemes: explicit and implicit. This section briefly reviews the theory of finite-difference migration.

The extrapolation equation of the two-dimensional wave  $P(x, z)$  is given by the following square-root equation (Claerbout, 1976, p. 202),

$$\frac{P(z, \bar{z})}{P(z, z)} = ik \left(1 + \frac{\partial_{xx}/k^2}{1 + \frac{\partial_{xx}/k^2}{\sqrt{1 + \frac{\partial_{xx}/k^2}{\sqrt{1 + \frac{\partial_{xx}/k^2}{\sqrt{1 + \partial_{xx}/k^2}}}}}}} \right)^{1/2} P(z, z) \quad (1)$$

where  $x$  is the horizontal distance,  $z$  is depth,  $k$  is the wave number, and  $P_z$  is the derivative of  $P$  with respect to  $z$ . Equation (1) represents a wave propagating in the one  $z$ -direction, i.e., positive or negative, and is called the one-way equation. The finite-difference migration uses an approximate one-way equation which is a rational approximation of equation (1) with respect to  $\partial_{xx}$ . An explicit approximation uses the Taylor series,

$$\frac{P(\bar{z})}{P(z)} = ik \left(1 + \frac{\partial_{xx}/2k^2}{1 + \frac{\partial_{xx}/2k^2}{1 + \frac{\partial_{xxx}/8k^4}{1 + \dots}}} \right) P(z) \quad (2)$$

In the wave extrapolation, wave field  $P$  and its  $x$ -derivatives are known, while the  $z$ -derivative  $P_z$  is unknown. Equation (2) represents the unknown  $P_z$

in terms of the known explicitly. Therefore, it is an explicit equation. The explicit scheme computes the  $x$ -derivatives either by a convolution in the space domain or by a multiplication in the wavenumber domain. By computing the right-hand side of equation (2), the wave at  $z + \Delta z$  is calculated by the following equation,

$$P(z + \Delta z) = P(z) + \Delta z \frac{\partial P(z)}{\partial z} + (\Delta z)^2 \frac{\partial^2 P(z)}{\partial z^2} / 2 + \dots \quad (3)$$

One of the advantages of the explicit scheme is that the accuracy of the  $x$ -derivative is easily improved, for example, by using a long difference operator.

The implicit scheme of the finite-difference migration approximates equation (1) as

$$\frac{P(\bar{z})}{P(z)} = ik \left(1 + \frac{\sum_j a_j \frac{\partial_{xx}^j}{1 + \sum_j b_j \frac{\partial_{xx}^j}{1 + \dots}}}\right) P(z) \quad (4)$$

The coefficients  $a_j$  and  $b_j$  may be obtained either by the continued-fractions method (Hildebrand, 1956, p. 406) or by the least-squares optimization method (Lee and Suh, 1985). A polynomial expression of equation (4) represents the unknown  $P_z$  in terms of the unknown  $P_{xxz}$ .

Therefore, equation (4) is an implicit equation. A solution method of equation (4) is developed by Ma (1981). In his method, equation (4) is divided into partial fractions as

$$\frac{P(\bar{z})}{P(z)} = ik \left(1 + \sum_j \frac{\alpha_j \frac{\partial_{xx}^j}{1 + \beta_j \frac{\partial_{xx}^j}{1 + \dots}}}\right) P(z) \quad (5)$$

The solution to equation (5) or, equivalently, to equation (4) may be obtained by solving the split equations separately. A split equation resembles the so-called 45-degree equation,

$$\frac{P(\bar{z})}{P(z)} = ik \frac{\alpha \frac{\partial_{xx}}{1 + \beta \frac{\partial_{xx}}{1 + \dots}}}{\alpha \frac{\partial_{xx}}{1 + \beta \frac{\partial_{xx}}{1 + \dots}}} P(z) \quad (6)$$

## MIGRATION TEST

It is generally believed that the finite-difference method is not accurate for steep-dip migration. Such a limitation is primarily due to the inaccurate dispersion relation of the one-way equation employed by the conventional method. In this section, a model experiment is given by both the explicit and implicit schemes. Figure 1 shows a synthetic reflector model with a homogeneous velocity of 3,000 m/sec. The reflector is asymmetrically W-shaped, the leftmost segment representing a 70-degree dip, and the rightmost segment representing a 60-degree dip, respectively. Two segments in the central part simulate a 45-degree dip.

In a homogeneous medium, the propagation angle of the normal ray, with respect to the z-axis, is the same as the dip angle. Whereas, the propagation angle is generally less than the dip angle because, normally, the velocity decreases as the ray propagates from the reflector to the surface.

Figure 2 is a synthetic zero-offset section and is computed by the phase-shift method (Gazdag, 1978). The grid intervals are  $\Delta x = \Delta z = 50$  m with the number of grid points in the x- and z-directions by 512 and 121, respectively. The time increment  $\Delta t$  is 50 msec and the number of time samples is 256.

Figure 3 shows three migrated sections of figure 2. The result is represented in (x, t) domain, i.e., time migration. Figure 3A is computed by the program MIGRATX of DISCO (Digicon's Interactive Seismic Computer) software. The program uses the explicit finite-difference algorithm and controls the difference error quite accurately. Lee and Suh (1985) studied the dispersion relations of explicit and implicit one-way equations and showed that the former is less accurate than the latter for the same order of approximation. In figure 3A, the 45-degree segments are properly migrated, but the steeper segments are still under-migrated. The under-migration is caused by the inaccurate dispersion relation of the explicit one-way equation.

Figures 3B and 3C are computed by the implicit method using the optimized second-order equation and the optimized fourth-order equation (Lee and Suh, 1985), respectively. The figures show remarkable differences. The effect of under-migration in figure 3A is progressively diminishing in figures 3B and 3C. However, the two figures show phase error in steeper segments caused by the numerical error in the derivative-to-difference approximation.

There are two methods of suppressing the numerical error. The one is by using a smaller sampling interval, and the other is by using a more elaborate difference scheme. Figure 4 shows a migrated result of figure 2 using the optimized fourth-order equation with a new sampling interval  $\Delta x = 12.5$  m. Changing the sampling interval can be accomplished by an interpolation. The result shows neither the under-migration nor the phase error. The 70-degree dip is successfully migrated.

Another criteria for evaluating the performance of these various migration schemes is to compare computing times. Table 1 summarizes the computing times for the migration together with its input parameters. The computation is done by a VAX 11/780 CPU and an AP 120B array processor.

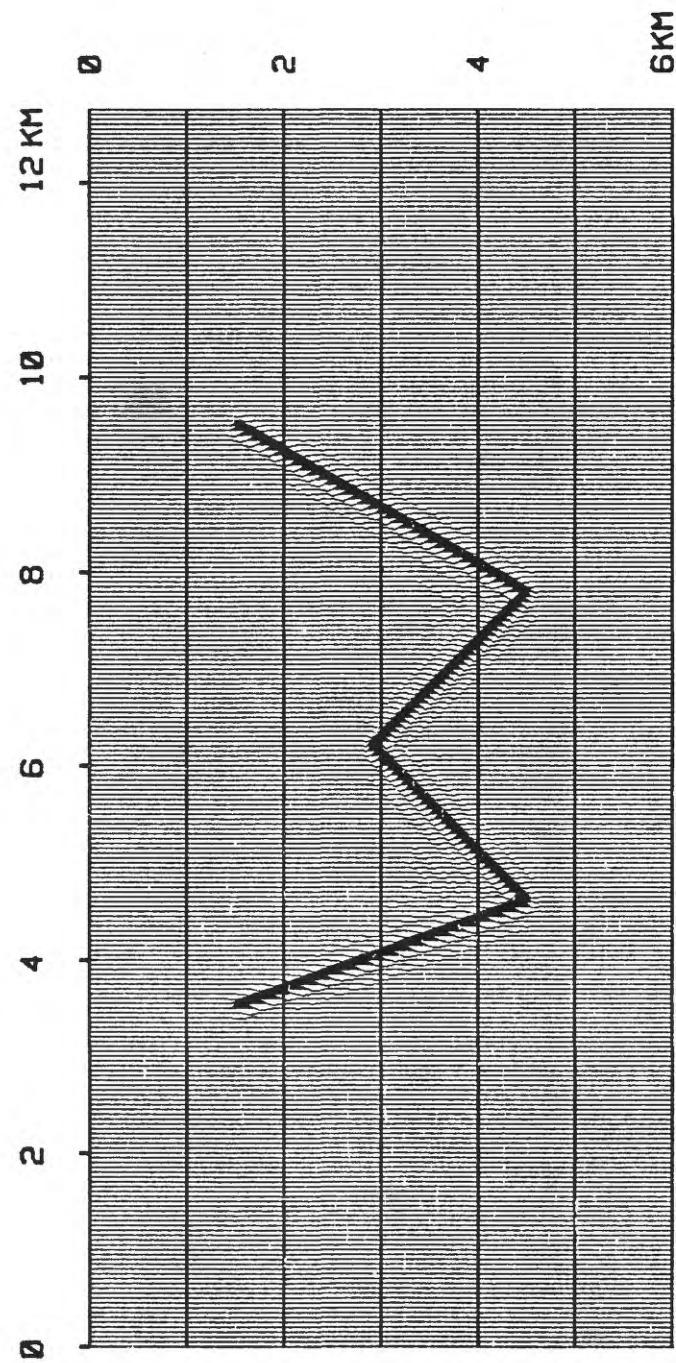
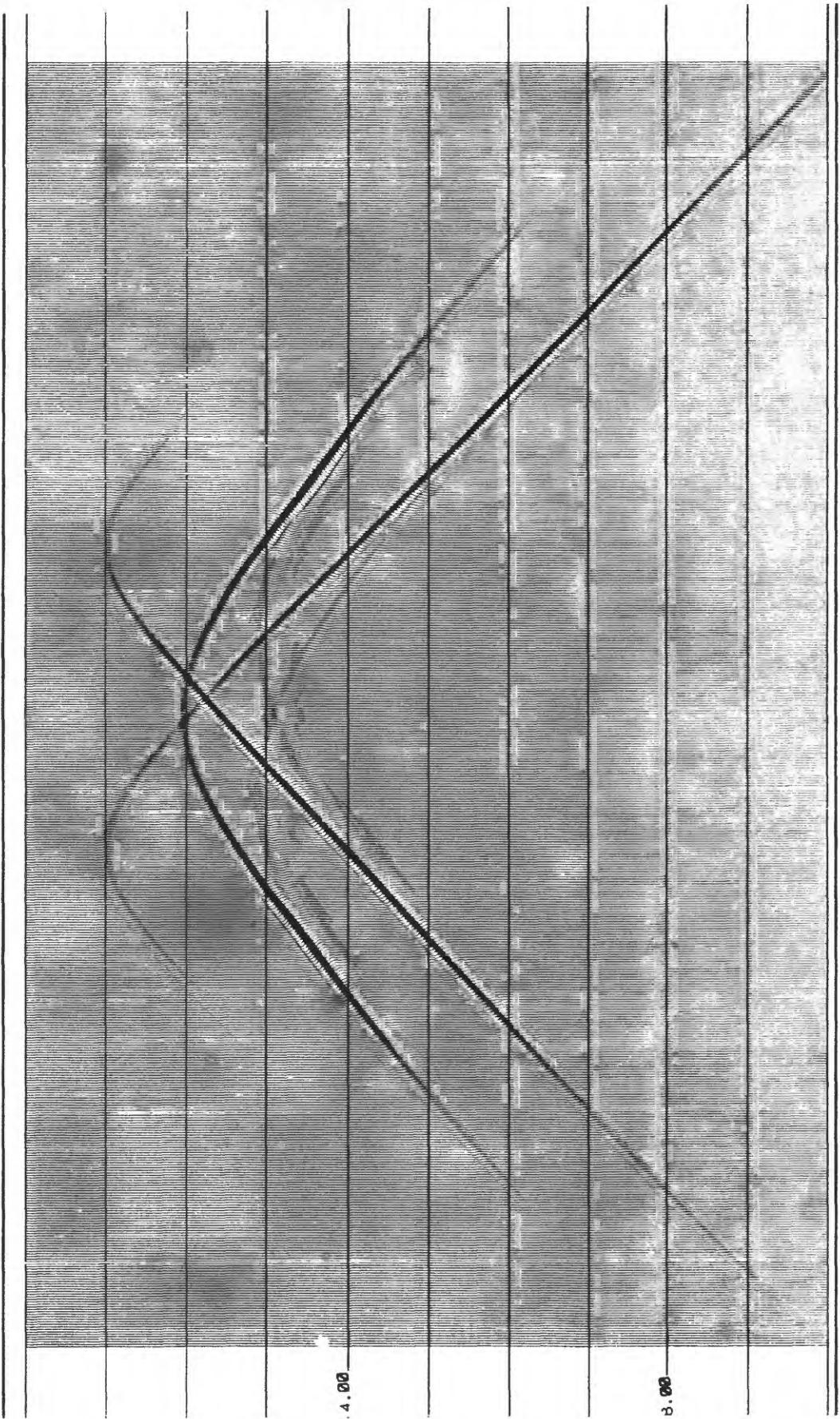


Figure 1---A W-shaped reflector model.

Figure 2.—Synthetic zero-offset section of figure 1.



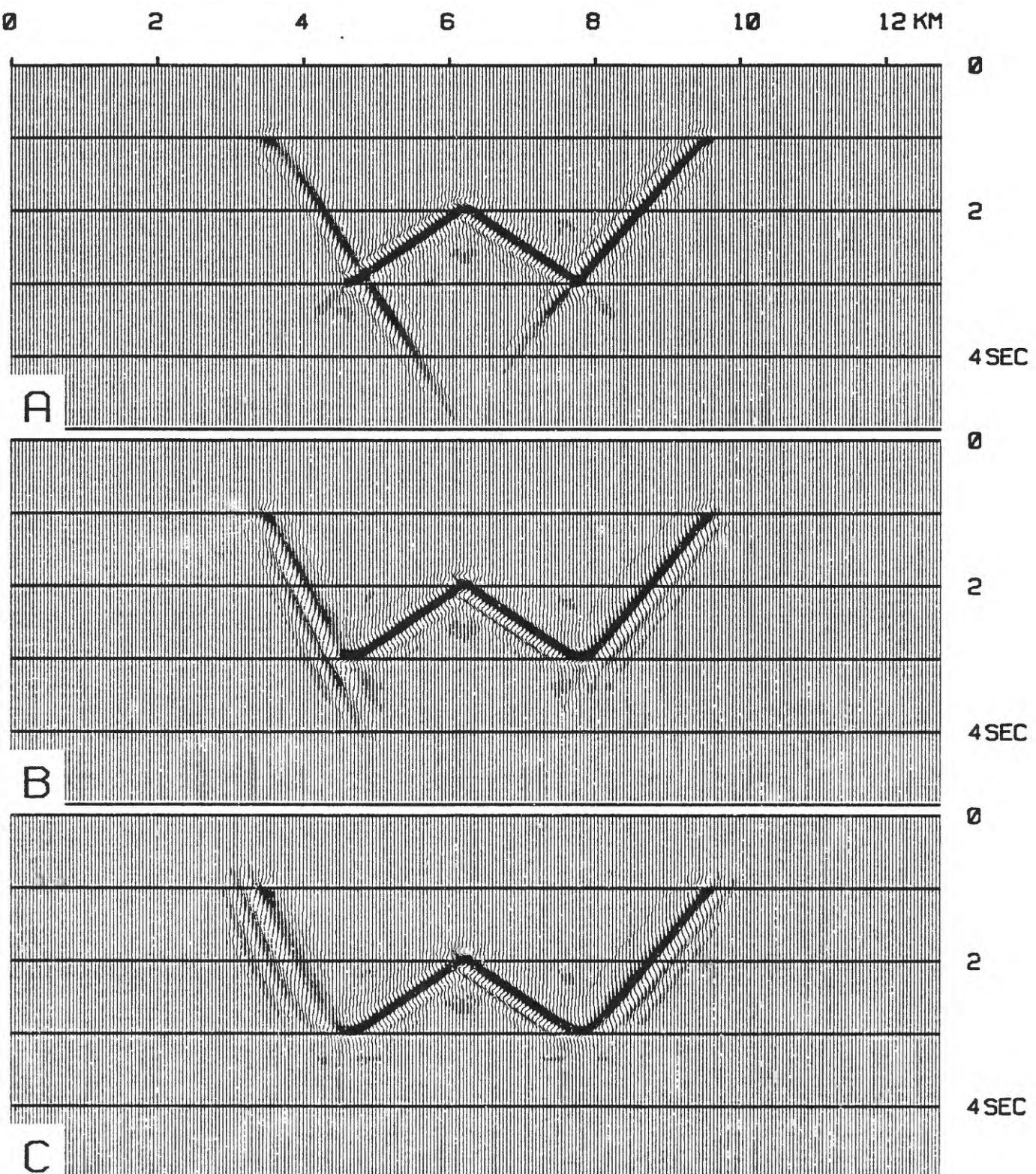


Figure 3.—Finite-difference migration of figure 2 by an explicit scheme (A), by an implicit second-order equation (B), and by an implicit fourth-order equation (C).

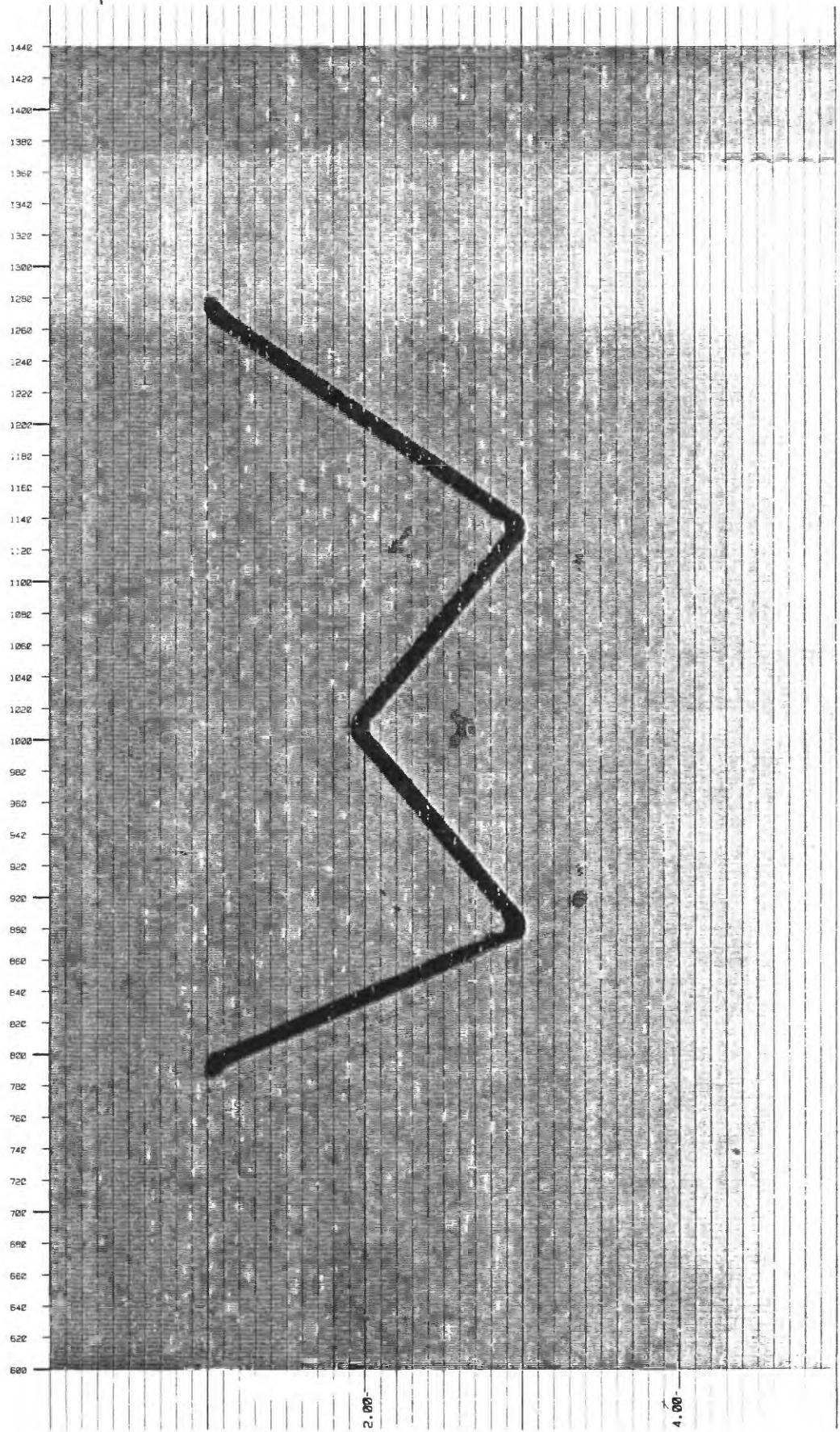


Figure 4.--Finite-difference migration of figure 2 by the implicit fourth-order equation with reduced trace interval.

The array processor has 65,536 words of memory and has a 167 nanoseconds cycle time. The first and second columns of the table show the figure name and the number of traces in the input data. The third column shows the sample rate of the input data in msec. The sample rates are different from that of the synthetic zero-offset section of figure 2, i.e., the inputs are interpolated in the time axis. The fourth column of the table shows the downward-continuation interval in msec. The fifth and sixth columns indicate the CPU time and the AP time, respectively, in seconds. The last column represents the number of page faults which greatly affects the computing time in a virtual memory system. Page faults can be reduced by increasing the size of a working set in a VAX computer. However, a common working set size was used for this comparison.

Table 1.--Run-time report on the finite-difference migrations

| Fig.<br>No. | No. of<br>traces | $\Delta t$<br>(msec) | $\Delta \tau$<br>(msec) | CPU time<br>(sec) | AP time<br>(sec) | Page<br>faults |
|-------------|------------------|----------------------|-------------------------|-------------------|------------------|----------------|
| 3A          | 512              | 4                    | 20                      | 1,247.24          | 3,855.13         | 107,057        |
| 3B          | 512              | 20                   | 20                      | 42.99             | 331.04           | 3,435          |
| 3C          | 512              | 20                   | 20                      | 43.12             | 514.40           | 3,454          |
| 4           | 2,048            | 10                   | 10                      | 15,917.66         | 5,049.24         | 1,153,956      |

There are two kinds of computing time in the migration. One is the time for wave-field extrapolation, and the other is the time for matrix transposition. The extrapolation requires a lot of arithmetic operations and is the task of the array processor. The AP time in table 1 is the time for extrapolation. On the other hand, the matrix transposition in migration requires a lot of physical memory and is accomplished by the CPU. The matrix transposition used most of the CPU time in table 1.

The AP time of the implicit scheme is generally less than that of the explicit scheme. The CPU time is highly dependent on the number of samples of the input data. Figure 4 takes a tremendous amount of CPU time for the matrix transpose, which is caused by the resampling in x-direction. This suggests that interpolation is not an effective method for the finite-difference scheme.

#### DERIVATIVE-TO-DIFFERENCE APPROXIMATION

One of the major concerns of the finite-difference method is to reduce the numerical dispersion error caused by the finite sampling interval. Decreasing the sampling interval is one of the accepted methods of reducing numerical error, but it is time-consuming and inefficient. Thus, more accurate derivative-to-difference approximations are desirable.

For a steep-dip migration, the apparent wavenumber in the x-direction increases according to the dip angle, thereby reducing the apparent wavelengths. As a result, the effect of numerical errors is more obvious in the steep-dip migration. Therefore, a more accurate derivative-to-difference approximation in the x-direction is required. Claerbout (1976) introduced an implicit difference approximation of the second derivative as

$$\tilde{\delta}_{xx}^{(2)} = \frac{1}{(\Delta x)^2} \frac{\delta_{xx}}{1 + \delta_{xx}/2} \quad (7)$$

where

$$\delta_{xx} f(x) = f(x+\Delta x) - 2f(x) + f(x-\Delta x).$$

Equation (7) is a rational approximation to the first two terms of the following series expression of  $\delta_{xx}$ .

$$\delta_{xx} = \frac{1}{(\Delta x)^2} \left( \delta_{xx} - \frac{1}{12} \delta_{xx}^2 + \frac{1}{90} \delta_{xx}^3 - \frac{1}{560} \delta_{xx}^4 + \dots \right). \quad (8)$$

The finite-difference formulation of equation (6) by equation (7) results in a tri-diagonal equation. The formulation by the first term of equation (8) also results in a tri-diagonal equation. Therefore, equation (7) takes little more computing time than the first term approximation but provides more accurate results.

The  $2n$ -th order approximation of  $\delta_{xx}$  may be written in the following rational form,

$$\tilde{\delta}_{xx}^{(2n)} = \frac{1}{(\Delta x)^2} \frac{\sum_{j=1}^n c_j \tilde{\delta}_{xx}^j}{1 + \sum_{j=1}^n d_j \tilde{\delta}_{xx}^j}. \quad (9)$$

The coefficients  $c_j$  and  $d_j$  may be found by converting equation (8) into the rational form. The accuracy of the approximation is observed in the wavenumber domain. The Fourier transform of  $\delta_{xx}$  is

$$\tilde{\delta}_{xx} = -k_x^2. \quad (10)$$

For the unit grid interval, the Fourier transform of  $\delta_{xx}^{(2n)}$  is

$$\tilde{\delta}_{xx}^{(2n)} = \frac{\sum_j c_j \tilde{\delta}_{xx}^j}{1 + \sum_j d_j \tilde{\delta}_{xx}^j} \quad (11)$$

where

$$\tilde{\delta}_{xx} = 2(\cos k_x - 1).$$

Therefore, the error  $E^{(2n)}$  of equation (9) is the difference between equation (11) and equation (10), i.e.,

$$E^{(2n)}(k_x) = \frac{\sum_j c_j \tilde{\delta}_{xx}^j}{1 + \sum_j d_j \tilde{\delta}_{xx}^j} + k_x^2. \quad (12)$$

Equation (12) is very similar to the dispersion error of the 2n-th order implicit equation (Lee and Suh, 1984). This suggests that coefficients  $c_j$  and  $d_j$  may be found by a least-squares method. Introducing a weighted error, i.e.,

$$\hat{E}^{(2n)}(\hat{k}_x) = \sum_{j=1}^n c_j \hat{\delta}_{xx}^j + \hat{k}_x^2 \left( 1 + \sum_{j=1}^n d_j \hat{\delta}_{xx}^j \right) \quad (13)$$

the least-squares method is reduced to the linear problem. Therefore, the coefficients are found by the minimization of the integral

$$J \triangleq \int_0^\phi \left[ \hat{E}^{(2n)}(\hat{k}_x) \right]^2 d\hat{k}_x \quad (14)$$

where  $\phi$  is the maximum wavenumber of the optimization. The wavenumber does not exceed the Nyquist wavenumber (180 degrees).

Table 2 shows the least-squares solution for fourth- and sixth-order approximation. The first column indicates the order of the approximation. The second column shows the maximum wavenumber of the optimization. The third and fourth columns show the estimated coefficients for the numerator and denominator of equation (9), respectively. The last column indicates the subscript of each coefficient.

Table 2.--Estimated coefficients of the difference operators in equation (9)

| Order | $\phi$ | $c_j$        | $d_j$        | j |
|-------|--------|--------------|--------------|---|
| 4     | 135    | 1            | .260 013 893 | 1 |
|       |        | .175 550 990 | .011 521 881 | 2 |
| 6     | 180    | 1            | .494 329 422 | 1 |
|       |        | .410 176 191 | .071 525 600 | 2 |
|       |        | .040 479 975 | .002 565 484 | 3 |

Figure 5 shows the amplitude spectra of the derivative operators. Graph A is the spectrum of the exact second-derivative operator. Graph B is the spectrum of the second-order approximate operator given in equation (7). Graphs C and D are the spectra of the fourth- and sixth-approximate operators. Figure 5 shows that graph B is acceptable up to about 50 percent of the total bandwidth, while graphs C and D are acceptable up to about 75 percent and 90 percent, respectively. This suggests that the conventional difference operator given in equation (7) may be replaced by the more accurate operators which are fourth- and sixth-order approximation of  $\partial_{xx}$ .

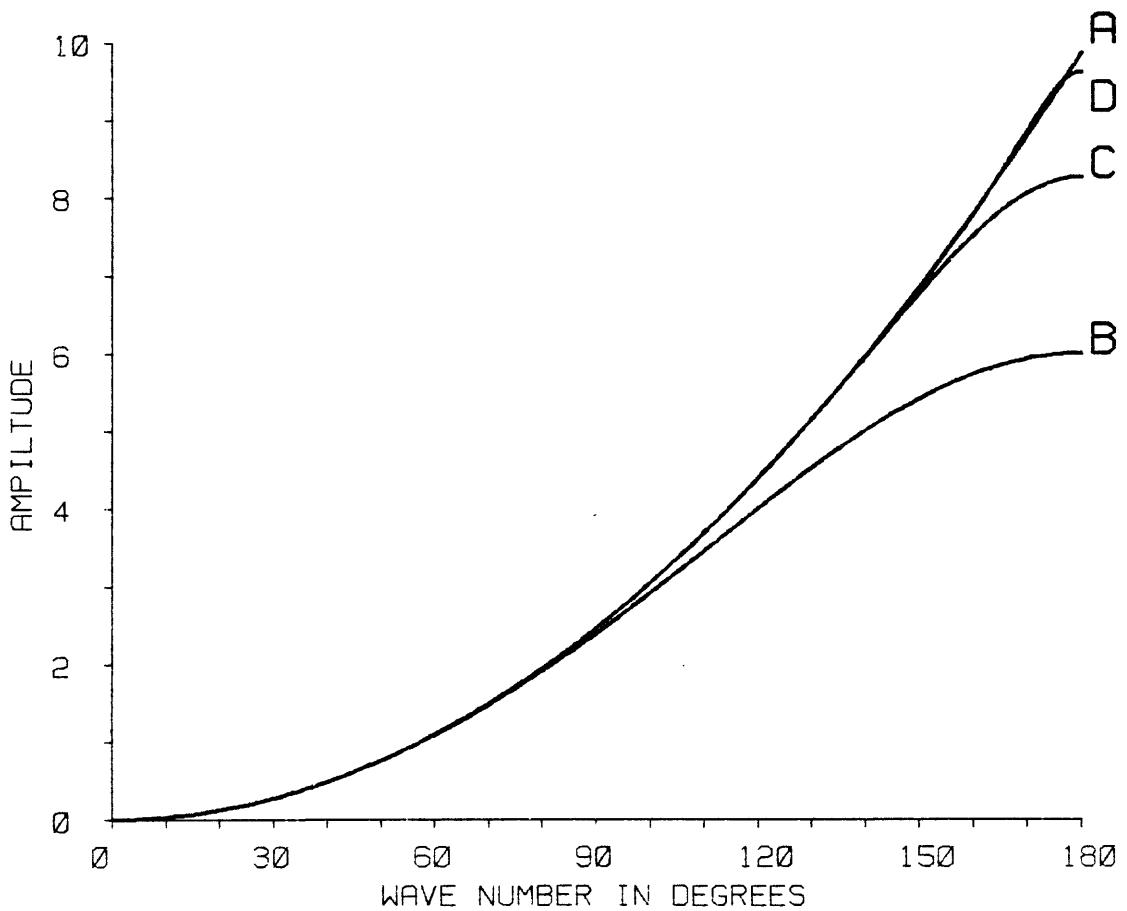


Figure 5.--Amplitude spectra of second-derivative operators:  
A is the exact second-derivative operator; B, a second-order  
approximate operator; C, a fourth-order approximate operator;  
and D, a sixth-order approximate operator.

The finite-difference formulation of equation (6) by high-order difference approximation does not result in a tri-diagonal equation, i.e., the fourth-order approximation gives a pentagonal equation and the sixth-order approximation gives a heptagonal equation. The general form of the heptagonal equation is

$$A_k T_{k-3} + B_k T_{k-2} + C_k T_{k-1} + D_k T_k + E_k T_{k+1} + F_k T_{k+2} + G_k T_{k+3} = H_k \quad (15)$$

for  $k = 3, 4, \dots, N-3$ . In the equation,  $A_k$  through  $H_k$  are known coefficients and  $T$  is the unknown. The solution to equation (15) employs an auxilliary equation of

$$T_k = P_k T_{k+3} + Q_k T_{k+2} + R_k T_{k+1} + S_k, \quad (16)$$

where the coefficients  $P_k$  through  $S_k$  are firstly found by comparing to equation (15). The unknown  $T_k$  are then computed by equation (16).

The number of arithmetic operations in the finite-difference method increases according to the order of the difference approximation. Let us assume that there are  $N$  input traces for the migration. The number of significant elements in the tri-diagonal matrix is approximately  $3N$ , while those in the pentagonal and heptagonal matrices are  $5N$  and  $7N$ , respectively. Therefore, the number of arithmetic operations in high-order difference approximation increases approximately by the same ratio.

Table 3 shows an estimated time report to migrate figure 2, by using the high-order difference approximations. The first column of the table shows the order of the optimized one-way equation. The second column shows the order of the difference approximations. The third, fourth and fifth columns show the number of traces, the sample rate ( $\Delta t$ ), and the downward-continuation interval ( $\Delta \tau$ ), respectively. The sixth and seventh columns show the estimated CPU time and AP time, respectively, in seconds. The CPU time is easily predictable from table 1. The AP times are computed by scaling the corresponding AP time by the factor of 1.67 or 2.33 depending on the order of the difference approximation.

Table 3.--Estimated time report on finite-difference migration of figure 2

| Order of one-way equation | Order of difference equation | No. of traces | $\Delta t$ (msec) | $\Delta \tau$ (msec) | CPU time (sec) | AP time (sec) |
|---------------------------|------------------------------|---------------|-------------------|----------------------|----------------|---------------|
| 2                         | 4                            | 512           | 20                | 20                   | 43             | 550           |
| 2                         | 6                            | 512           | 20                | 20                   | 43             | 770           |
| 4                         | 4                            | 512           | 20                | 20                   | 43             | 833           |
| 4                         | 6                            | 512           | 20                | 20                   | 43             | 1,170         |
| 4                         | 6                            | 512           | 20                | 10                   | 43             | 2,340         |

## CONCLUSIONS

Two finite-difference migration schemes, i.e., the explicit and the implicit, were tested using a synthetic model with a maximum dip of 70 degrees. The tests showed that the explicit scheme is subject to the dip limitation of the one-way equation while the implicit scheme is subject to the numerical dispersion. The numerical dispersion can be suppressed by the interpolation of the input data, but it takes a tremendous amount of CPU time in the matrix transposition. A new method of controlling the numerical dispersion effect is presented in this paper which employs optimized high-order difference approximations. Because this suggested method is computatively more efficient than the conventional method, it is worth considering for implementing steep-dip migration.

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#### APPENDIX: ARRAY PROCESSOR VERSION OF THE MIGRATION PROGRAM

Migration is one of the most time-consuming processes in seismic processing. Even by use of a modern high-speed computer that can compute almost one million floating-point operations in a second, the process takes hours of computing time. By introducing the concept of the array processor (AP), the time problem of migration is greatly reduced. The array processor computes multiple operations simultaneously. The operations are further divided into several steps and computed in a pipe-line mode; thereby millions of arithmetic additions and multiplications can be calculated within a second. This version of the migration program uses practically all the resources of the array processor. The input goes to the array processor, and the output, which is the migrated result, comes from the array processor. The I/O operation between the CPU and AP is minimized. This is accomplished by rewriting the main algorithm into AP assembler language.

The current version of the program supports time migration by the optimized one-way wave equations from the second-order to the tenth-order equation (Lee and Suh, 1985), and by the optimized second-order difference operator. Since the migration is a part of a stream of other seismic processes, the program is designed to handle the output of other processes. Therefore, the program is further converted to be compatible with one of the most commonly used software packages in the exploration industry, the DISCO (Digicon's Interactive Seismic Computer). The following describes the program in the form of the DISCO user's manual as well as the source statements. The name of the program is MIGRHI.

MIGRHI

|          |          |        |        |        |        |
|----------|----------|--------|--------|--------|--------|
| "*CALL"  | "MIGRHI" | DX     | ORDER  | DAMP   | ADDFAC |
| "KEY"    | KEYNAM   | KYV1ST | KYVLST | KYVINC |        |
| "LAYER"  | DTAU     | ETIME  | F1     | F2     |        |
| "VELOCT" | VTYPE    | VIDENT |        |        |        |

C  
C PARAMETER DESCRIPTIONS

C LU: LOWER LIMIT  
C UL: UPPER LIMIT  
C DEF: DEFAULT VALUE

|         |             |   |   |      |        |
|---------|-------------|---|---|------|--------|
| "*CALL" | "MIGRHI"    | DX  | ORDER   | DAMP | ADDFAC |
| DX      |             | FLOATING PT   | 'HORIZONTAL TRACE SPACING IN FEET<br>OR METERS' |      |        |
| LL :    |             |   |   |      |        |
| UL :    |             |   |   |      |        |
| DEF:    |             |   |   |      |        |
| ORDER   | INTEGER     |   | 'ORDER OF THE ONEWAY EQUATION.'                 |      |        |
| LL :    | '2'         |   |   |      |        |
| UL :    | '10'        |   |   |      |        |
| DEF:    | '2'         |   |   |      |        |
| DAMP    | FLOATING PT | 'FACTOR OF THE NUMERICAL DIP-FILTER.<br>DAMP=   |   |      |        |
| LL :    | '0'         |   |   |      |        |
| UL :    | '5.'        |   |   |      |        |
| DEF:    | '0'         |   |   |      |        |
| ADDFAC  | FLOATING PT | 'FACTOR OF DATA EXTENSION TO PREVENT<br>THE WRAP-AROUND EFFECT OF F-X-Z<br>MIGRATION ALGORITHM' |   |      |        |
| LL :    | '1.'        |   |   |      |        |
| UL :    | '2.'        |   |   |      |        |
| DEF:    | '1.'        |   |   |      |        |

KEY

|        |         |        |  |        |  |
|--------|---------|--------|--|--------|--|
| "KEY"  | KEYNAM  | KYV1ST | KYVLST   | KYVINC |  |
| KEYNAM | C* 8    |        | 'HEADER ENTRY NAME OF VELOCITY CONTROL<br>POINTS.' |        |  |
| DEF:   | 'COP'   |        |  |        |  |
| KYV1ST | INTEGER |        | 'FIRST KEY VALUE OF THE TRACE TO BE<br>MIGRATED.'  |        |  |

```
LL :  
UL :  
DEF :  
KYVLST    INTEGER      'LAST KEY VALUE OF THE TRACE TO BE  
                         MIGRATED.'  
LL :  
UL :  
DEF :  
KYVINC    INTEGER      'INCREMENT OF THE KEY VALUE.  
                         NOTE:  
                         IF(KYVINC.GT.1) DX MUST BE SET TO  
                         DX*KYVINC.  
LL : '1'  
UL :  
DEF : '1'
```

#### LAYER

```
"LAYER"   DTAU      ETIME      F1          F2  
DTAU       FLOATING PT  'LAYER THICKNESS IN MSEC.'  
LL :  
UL :  
DEF : '40.'  
ETIME     FLOATING PT  'END MIGRATION TIME IN MSEC.'  
LL :  
UL :  
DEF :  
F1        FLOATING PT  'LOWER FREQUENCY LIMIT IN HERZ.'  
LL : '0'  
UL :  
DEF : '0'  
F2        FLOATING PT  'UPPER FREQUENCY LIMIT IN HERZ.  
                         NOTE:  
                         TO SAVE COMPUTING TIME, SUPPLY SIGNAL  
                         BAND ONLY. THE BAND MAY BE LOCATED BY  
                         SPECTRUM ANALYSIS OF INPUT DATA.'  
LL :  
UL :  
DEF :  
VIDENT    C*16         'VELOCITY IDENT NAME DEFINED TO SEISD
```

#### VELOCT

```
"VELOCT"  VTYPE      VIDENT  
VTYPE     INTEGER      'VELOCITY TYPE:  
                         0 = RMS VELOCITY  
                         1 =  
LL : '0'  
UL : '1'  
DEF : '0'  
VIDENT    C*16         'VELOCITY IDENT NAME DEFINED TO SEISD  
DEF :  
C
```

C. THE FOLLOWING DATA DECK WAS USED TO COMPUTE FIGURE 3B IN THE TEXT

C

\*JOB KOREA L8053  
\*ALT MIGRHI [LEE.MIGRHI]  
\*CALL DSKRD [RIFLE.NIDS]HYBRIDS  
\*CALL RESAMP 20.0  
\*CALL MIGRHI 50 2  
KEY CHAN 1 512 1  
LAYER 20 5000 1. 10.  
VELOCT 1 V3000  
\*CALL SEC PLOT LR VAWG 20 1.25  
LABEL CHAN 20  
TITLE FIG. 3B.  
TRANGE 0 5000  
TIMING 2 1 0 0  
SETAMP PEAK FIRST  
GAIN 2  
MAXTR 512  
\*END

PROGRAM    \*\*    M I G R A T I    \*\*    VERSION 1M-8    NOV/24/84

## FINITE-DIFFERENCE WAVE EQUATION TIME-MIGRATION

## ALGORITHM DESCRIPTION

1. THE EXACT SQUARE-ROOT ONE-WAY EQUATION IS OPTIMIZED BY A ONEWAY EQUATION, THE ORDER OF WHICH MAY BE DETERMINED BY THE USER. FIVE DIFFERENT ONEWAY EQUATIONS ARE SUPPORTED, I.E., SECOND, FOURTH, SIXTH, EIGTH, AND TENTH ORDER EQATIONS, THE DISPERSION RELATION OF WHICH ARE ACCURATE WITHIN 1 % LIMIT FOR THE PROPAGATION ANGLE OF UP TO 65, 80, 87, 89, AND 90 DEGREES RESPECTIVELY.
  2. THE OPTIMIZED ONE-WAY EQUATION IS EXPANDED TO THE FINITE-DIFFERENCE EQUATIONS EMPLOYING AN IMPLICIT DIFFERENCE EQUATION, WHICH IS BASICALLY SAME AS CLAERBOUT' METHOD. BUT THE APPROXIMATION OF X-DERIVATIVE TO DIFFERENCE IS IMPROVED IN THIS VERSION, SO THAT IT CAN HANDLE UP TO THE QUATER WAVELENGTH.
  3. ANOTHER REMARK GOES ON THE BOUNDARY CONDITION, WHICH, PRACTICALLY, IS TRANSPARENT AT THE BOUNDARY. THEREFORE YOU DO NOT HAVE TO ADD ZERO-TRACES AT THE BOUNDARY.
  4. REFER TO LEE AND SUH, 1984, FOR THE DETAIL.

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## **EDIT-PHASE PROGRAMS**

```
SUBROUTINE MIGRHI_EDITP
INCLUDE    'MONFORT/NOLIST'
INCLUDE    'MIGRHI.CMB/NOLIST'
```

## CONTENTS OF MIGRHI.CMB

```
COMMON /MIGRHI_BLK1/ NX, NZ, NT, DX, DZ,  
+ DDT, NFFT, NTOUT, NX1, NZ1, NX2, NW, NW2,  
+ DW, SW0, WIMAG, AMIMAG, NESTEQ, NZMINR, NZGRND, NWFOLD,  
+ NWGLOB, JWSTAT, EXTBUF(18,5)
```

**SUBROUTINE MTG8HT GETCBP**

```

C READ CONTROL PARAMETERS
C
INCLUDE      'MONFORT/NOLIST'
INCLUDE      'MIGRHI.CMB/NOLIST'
CHARACTER*8   NAMTAB (3)
DATA          NAMTAB / 'KEY', 'LAYER', 'VELOCT' /
DATA          NUMTAB / 3 /
CALL SETGBL (NCARDS)
DX      = FPARM ('DX',     000, 0., 0., 0.)
NESTEQ = IPARM ('ORDER',  111, 2, 10, 2) / 2
WPCNT  = FPARM ('DAMP',   111, 0., 5., 0.)
FFTFACT = FPARM ('ADDFAC', 111, 1., 2., 1.)

C GET LIST PARAMETERS
C
DO 400 K = 1, NCARDS
CALL NXTLST (NAMTAB, NUMTAB, INDEX, NREP)
GO TO (100, 200, 300), INDEX

C GET KEY PARAMETERS
C
100 KEYNAM = CPARM ('KEYNAM', 001, 'CDP')
KYV1ST = IPARM ('KYV1ST', 000, 0, 0, 0 )
KYVLST = IPARM ('KYVLST', 000, 0, 0, 0 )
KYVINC = IPARM ('KYVINC', 101, 1, 0, 1 )
GO TO 400

C READ LAYER PARAMETERS
C
200 DZ      = FPARM ('DTAU', 001, 0., 0., 40.)
STIME   = 0.
ETIME   = FPARM ('ETIME', 000, 0., 0., 0. )
SW1     = FPARM ('F1',    101, 0., 0., 0. )
SW2     = FPARM ('F2',    000, 0., 0., 0. )
GO TO 400

C READ VELOCITY CARD
C
300 IVTYPE = IPARM ('VTYPE', 111, 0, 1, 0 )
VIDENT = CPARM ('VIDENT', 000, 00000 )
400 CONTINUE

C NOW CHECK THE PARAMETERS      * * * * * * * * * * *
C
IF (DX .LE. 0.)                      GO TO 800
NESTEQ = MAX0 (1, MIN0 (5, NESTEQ))
WPCNT  = AMAX1 (0., AMIN1 (5., WPCNT))
FFTFACT = AMAX1 (1., AMIN1 (2., FFTFACT))
IF (THDRGET(KEYNAM, KEYLEN, KEYFMT, KEYIND, 'E') .EQ. 0)
+                               GO TO 801
NX      = (KYVLST - KYV1ST) / KYVINC + 1
IF (NX .LT. 2)                       GO TO 802
DOT    = DT / 1000.
NZMINR = DZ / DOT + 0.5
DZ     = DOT * NZMINR

```

```

NT      = LENGTH
TMAX   = DOT * (NT - 1)
ETIME  = AMINI (ETIME, TMAX)
NTOUT  = LENGTH
NZ     = ETIME / DZ + 1.5
DOT    = DOT / 1000.
DZ     = DZ / 1000.
FNYQ   = 0.5 / DOT
IF (SW1.GE.SW2 .OR. SW2.GT.FNYQ) GO TO 803

C
C COMPUTE NTFFT
C

NUMBER = NT * AMAX1 (1., AMIN1 (2., FFTFAC))
NTFFT = 1
DO 10 K = 1, 15
NTFFT = NTFFT + NTFFT
IF (NTFFT .GE. NUMBER) GO TO 12
10 CONTINUE
STOP  'ERROR IN NTFFT'
12 CONTINUE
PI     = ACOS (-1.)
DW    = 2. * PI / (NTFFT * DDT)
SW1   = 2. * PI * SW1
SW2   = 2. * PI * SW2

C
IW1    = SW1 / DW + 1.5
IW4    = SW2 / DW - 1.5
NW    = IW4 - IW1 + 1
SW0   = DW * IW1
SWMAJ = (SW1 + SW2) / 2.
WIMAG = WPCNT * SWMAJ / 100.
RETURN

C
C ERROR TERMINAL SECTION
C

800 STOP  'ERROR 800: DX'
801 STOP  'ERROR 801: KEYNAM'
802 STOP  'ERROR 802: KEYVAL'
803 STOP  'ERROR 803: F1/F2'
END

C
C
C SUBROUTINE MIGRHI_EXTCON
C
C INITIALIZE THE EXTRAPOLATION BUFFER
C

INCLUDE  'MONFORT/NOLIST'
INCLUDE  'MIGRHI.CMB/NOLIST'
DIMENSION DFCBUF(2)
DOUBLE PRECISION A(15), B(15)
DATA    MODE / 1 /
C
DATA    A(1) / 0. 376 369 527 234 052 / ! 65

```

```

C      DATA    B(1) / 0. 478 242 059 603 743 / ! 80
C      DATA    A(2) / 0. 873 981 642 171 890 / ! 80
C      DATA    B(2) / 0. 040 315 156 988 852 / ! 80
C      DATA    A(3) / 0. 222 691 982 666 100 / ! 80
C      DATA    B(3) / 0. 457 289 565 835 625 / ! 80
C      DATA    A(4) / 0. 972 926 131 694 782 / ! 87
C      DATA    B(4) / 0. 004 210 419 911 239 / ! 87
C      DATA    A(5) / 0. 744 418 058 525 258 / ! 87
C      DATA    B(5) / 0. 081 312 882 016 760 / ! 87
C      DATA    A(6) / 0. 150 843 924 026 968 / ! 87
C      DATA    B(6) / 0. 414 236 604 654 513 / ! 87
C      DATA    A(7) / 0. 991 834 774 675 097 / ! 89
C      DATA    B(7) / 0. 000 737 959 542 660 / ! 89
C      DATA    A(8) / 0. 911 282 437 100 351 / ! 89
C      DATA    B(8) / 0. 016 329 891 492 279 / ! 89
C      DATA    A(9) / 0. 602 498 780 802 238 / ! 89
C      DATA    B(9) / 0. 120 110 756 314 730 / ! 89
C      DATA    A(10) / 0. 102 624 305 081 323 / ! 89
C      DATA    B(10) / 0. 362 806 692 332 044 / ! 89
C      DATA    A(11) / 0. 997 370 236 438 328 / ! 90
C      DATA    B(11) / 0. 000 153 427 175 533 / ! 90
C      DATA    A(12) / 0. 964 827 991 878 123 / ! 90
C      DATA    B(12) / 0. 004 172 967 255 246 / ! 90
C      DATA    A(13) / 0. 824 918 564 779 961 / ! 90
C      DATA    B(13) / 0. 033 860 917 808 142 / ! 90
C      DATA    A(14) / 0. 483 340 757 434 262 / ! 90
C      DATA    B(14) / 0. 143 798 075 648 762 / ! 90
C      DATA    A(15) / 0. 073 588 212 879 826 / ! 90
C      DATA    B(15) / 0. 318 013 812 535 422 / ! 90
C      DATA    DFCBUF / 1. 000 000 000, 0. 124 600 000 /
C      C
C      IF (MODE .GE. 0)          THEN      ! MIGRATION MODE
C          DX      = ABS (DX)
C          DZ      = ABS (DZ)
C      ELSE                      ! MODELING MODE
C          DX      = -ABS (DX)
C          DZ      = -ABS (DZ)
C      ENDIF
C
K0      = (NESTEQ * (NESTEQ - 1)) / 2
DO 10 K = 1, NESTEQ
AA      = 1. / A(K+K0)
BB      = AA * B(K+K0)
CALL MIGRHI_CFBNDG (EXTBUF(17,K), A(K+K0), B(K+K0))
10    CALL MIGRHI_CFINTE (EXTBUF(1,K), AA, BB, DFCBUF(1), DFCBUF(2))
      RETURN
      END

```

```

SUBROUTINE MIGRHI_CFBNDG (BCOF, A, B)
C
C
C   GENERATE THE TRANSPARENT BOUNDARY COEFFICIENTS AS
C
C   B*XX / (1 + A*XX) === BB*Y / (1 + AA*Y), WHERE
C       XX      = DERIV(X)**2 / M, AND
C       Y       = I * DERIV(X)/ M.
C   BCOF(1) = AA
C   BCOF(2) = BB
C
C
DIMENSION BCOF(2)
LOGICAL VIRGIN
DATA     VIRGIN / .TRUE. /
C
C
COMPUTE INTEGRALS (S2 THRU S6)
C
IF (VIRGIN) THEN
    VIRGIN = .FALSE.
    S2    = ATAN (1.)
    S4    = S2 * 0.75
    S6    = S4 * 5. / 6.
    S3    = 2. / 3.
    S5    = S3 * 0.8
ENDIF
C
C
COMPUTE NORMAL EQUATION COEFFICIENTS
C
A11    = B * B * S6
A12    = B * (S4 - A * S6)
A22    = S2 - 2.*A*S4 + A*A*S6
B1     = B * B * S5
B2     = B * (S3 - A * S5)
C
C
SOLVE THE EQUATION
C
DETERM = 1. / (A11 * A22 - A12 * A12)
AA     = DETERM * (B1 * A22 - B2 * A12)
BB     = DETERM * (B2 * A11 - B1 * A12)
C
C
CONVERT COEFFICIENTS IN OLD FORMAT, I.E.,
C
C
BOLD(AA,BB) = Y / (AA + BB*Y)
C
BCOF(1) = 1. / BB
BCOF(2) = AA / BB
RETURN
END
SUBROUTINE MIGRHI_CFINTI (CFM, B, C, DFCNMR, ALPHA)
C
C
INITIALIZE DIFFERENCE COEFFICIENTS OF THE MAIN REGION
SEE P42R EQ. (6).
C
INCLUDE 'MIGRHI.CMB/NOLIST'
C
DIMENSION CFM(16)

```

```

C
BETA      = ALPHA   - 0.5
F1        = (B + B) * DX * DX / DFCNMR
F2        = (F1 + F1) * AMIMAG
G2        = C * DZ
G1        = -G2 * WIMAG

C
CALL MIGRHI_CFINTE (CFM(1), F1, F2, G1, G2, ALPHA, -1., -1.)
CALL MIGRHI_CFINTE (CFM(5), F1, F2, G1, G2, ALPHA, -1., 1.)
CALL MIGRHI_CFINTE (CFM(9), F1, F2, G1, G2, BETA, 2., 2.)
CALL MIGRHI_CFINTE (CFM(13), F1, F2, G1, G2, BETA, 2., -2.)
RETURN
END

SUBROUTINE MIGRHI_CFINTE (COF, F1, F2, G1, G2, ALPHA, R1, R2)
C
C DO THE COMPUTATION FOR CFINT2-INTERIOR
C
DIMENSION COF(4)
C
COF(1) = R1 + R1 + R2 * G1
COF(2) = ALPHA * R1 * F1
COF(3) = R2 * G2
COF(4) = R1 * F2 * ALPHA
RETURN
END

SUBROUTINE MIGRHI_SKIPCD (LDEV, NCARD)
DO 10 ICARD = 1, NCARD
10 READ (LDEV, 50)
50 FORMAT (1X)
RETURN
END

SUBROUTINE MIGRHI_VELGET (VIBAR, TIME, VELO)
C
C GET AND RESERVE THE VELOCITY INFORMATION
C
INCLUDE     'MONFORT/NOLIST'
INCLUDE     'MIGRHI.CMB/NOLIST'
C
DIMENSION TIME(2), VELO(2)
CHARACTER*50 FILDFN
CHARACTER*8  LINE
C
CALL INFOGET ('LINE', LINE)
CALL FNAMGET (LINE, 'VELDEFN', VIDENT, FILDFN, IFSTAT)
IF (IFSTAT .NE. HERR$OK) STOP  'CAN NOT FIND VIDENT'
CALL GETIOU (LDVDFN)
OPEN (UNIT = LDVDFN, FILE = FILDFN, TYPE = 'OLD', READONLY)
OPEN (UNIT = LDVVEL, FILE = FILVEL, TYPE = 'SCRATCH',

```

```

+      FORM = 'UNFORMATTED')
C
CALL MIGRHI_SKIPCD (LDVDFN, 2)
READ (LDVDFN, 50) NCNTRL
CALL MIGRHI_SKIPCD (LDVDFN, 29)
DO 20 ICNTRL = 1, NCNTRL
READ (LDVDFN, 50) ICHECK
C     IF (ICHECK .NE. 1)
C +      STOP      'VELOCITY NOT CORRECTLY DEFINED'
READ (LDVDFN, 50) NPAIR, JCNTRL
CALL MIGRHI_SKIPCD (LDVDFN, 17)
DO 10 I = 1, NPAIR
10   READ (LDVDFN, 52) TIME(I), VELO(I)
VALKEY = JCNTRL
20   WRITE (LDVVEL) VALKEY, NPAIR, (TIME(I), VELO(I), I = 1, NPAIR)
VIBAR = 1. / VELO(1)
CLOSE (LDVDFN)
CALL RELIOU (LDVDFN)
RETURN
50   FORMAT (I9)
52   FORMAT (2F14.1)
END

```

```

C * * * * *
C
C PROCESSING PHASE PROGRAMS
C * * * * *
C
C SUBROUTINE MIGRHI_PROCP (TRACE, THDR, IFLAG)
C
INCLUDE 'MONFORT/NOLIST'
INCLUDE 'MIGRHI.CMB/NOLIST'
LOGICAL NOMIGR
DATA NOMIGR / .TRUE. /
C
C
C
IF (PHASE .EQ. PH$PROC) THEN
  CALL MIGRHI_TM2FRQ (RCORE(FWACOR), TRACE, THDR)
  IFLAG = FLG$MULTI
ELSE
  IF (NOMIGR) THEN
    CALL MIGRHI_TM2FRF (RCORE(FWACOR))
    CALL MIGRHI_DOMIGR
    NOMIGR = .FALSE.
  ENDIF
  CALL MIGRHI_OUTPUT (TRACE, THDR, IFLAG, RCORE(FWACOR+NX))
ENDIF
RETURN
END

```

SUBROUTINE MIGRHI\_DOMIGR

C

```

C      DO THE MAJOR MIGRATION
C
C      INCLUDE      'MONFORT/NOLIST'
C      INCLUDE      'MIGRHI.CMB/NOLIST'
C
C      INITIALIZE THE PARAMETERS AND EDIT VELOCITY DATA.
C
NX1      =  NX - 1
NZ1      =  NZ - 1
NX2      =  NX + NX
MXNXNZ   =  MAXO (NX, NZ1)
I1       =  MXNXNZ + FWACOR
I2       =  MXNXNZ + I1
I3       =  MXNXNZ + I2
I4       =  MXNXNZ + I3
CALL MIGRHI_VELEDT (RCORE(I1), RCORE(I2), RCORE(I3),
+      RCORE(I4), RCORE(FWACOR))
C
C      GET FREQUENCY DATA IN MUX-FORMAT.
C
IWCPU   =  FWACOR + NX * (NTOUT + 1)
CALL MIGRHI_GETMXC (LDVFRQ, RCORE(IWCPU), NX, NW)
CLOSE (LDVFRQ)
CALL RELIOU (LDVFRQ)
C
C      FOLD THE FREQUENCY AND DO MIGRATION.
C
NWGLOB   =  NW
NWMINR   =  (MAXAPS - 201 - 11 * NX) / (3 + NX2)
NWFOLD   =  (NWGLOB - 1) / NWMINR + 1
CALL APEX_APINIT (0, 0, ISTAT)
DO 10 IWFOLD = 1, NWFOLD
JWSTAT   =  (IWFOLD - 1) * NWMINR + 1
NW       =  MINO (NWMINR, NWGLOB - JWSTAT + 1)
NW2      =  NW + NW
CALL MIGRHI_PUTCON (RCORE(FWACOR))
CALL APEX_APPUT (RCORE(IWCPU+(JWSTAT-1)*NX2), IQBUF, NX2*NW, 2)
CALL APEX_APWD
IF (IWFOLD .EQ. 1) THEN
    CALL MIGRHI_MLTST1 (RCORE(FWACOR), RCORE(FWACOR + NX))
ELSE
    CALL MIGRHI_MLTSTP (RCORE(FWACOR), RCORE(FWACOR + NX))
ENDIF
10      CONTINUE
CALL APEX_APRLSE
CLOSE (LDVVEL)
CALL RELIOU (LDVVEL)
RETURN
END

SUBROUTINE MIGRHI_PUTCON (BUF)
C
C      PUT MIGRATION CONSTANTS TO THE ARRAY-PROCESSOR.
C
INCLUDE      'MIGRHI.CMB/NOLIST'

```

```

DIMENSION BUF(200)
C
C      AP ADDRESS DEFINITIONS
C
LMIGCN = 200
IDZ = 1
ISWO = IDZ + 5
IDW = ISWO + 1
IDDT = IDW + 1
IEXTBF = 14
IWBUF = 201
ISBUF = IWBUF + NW
NW2 = NW + NW
IVBUF = ISBUF + NW2
IAMBF = IVBUF + NX
IAOLD = IAMBF + NX2
IQBUF = IAOLD + NX2 * 4
C
C      PUT CONSTANTS TO THE AP.
C
BUF(IDZ) = DZ
BUF(IDZ+1) = WIMAG
BUF(IDZ+2) = B + B
BUF(IDZ+3) = A * DX * NESTDF ! RESERVED
BUF(IDZ+4) = AMIMAG ! RESERVED
C
BUF(ISWO) = SWO + (JWSTAT - 1) * DW
BUF(IDW) = DW
BUF(IDDT) = DDT
C
DO 10 I = 1, NESTEQ
EXTBUF(17,I) = DX * EXTBUF(17,I)
10 EXTBUF(18,I) = 2. * EXTBUF(18,I)
CALL MIGRHI_MOVE (90, EXTBUF, BUF(IEXTBF))
CALL APEX_APPUT (BUF, IDZ, LMIGCN, 2)
CALL APEX_APWD
C
CALL FPS_VRAMP (ISWO, IDW, IWBUF, 1, NW)
CALL FPS_VSMUL (IWBUF, 1, IDDT, IQBUF, 1, NW)
CALL FPS_VCOS (IQBUF, 1, ISBUF, 2, NW)
CALL FPS_VSIN (IQBUF, 1, ISBUF+1, 2, NW)
CALL APEX_APWR
RETURN
END

SUBROUTINE MIGRHI_MLTSTP (VBUF, BUF)
C
C      MULTISTEP EXTRAPOLATION AND IMAGING.
C
INCLUDE 'MIGRHI.CMB/NOLIST'
DIMENSION VBUF(NX)
C
REWIND LDVVEL
CALL MIGRHI_IMAGEI
CALL MIGRHI_IMAGEZ (VBUF, BUF)

```

```

DO 10 IZ = 1, NZ1
READ (LDVVEL) VBUF
CALL APEX_APPUT (VBUF, IVBUF, NX, 2)
CALL APEX_APWD
CALL MIGRHI_FPS_EXTPOL (NX,IAMBF,IAOLD,IQBUF,IWBUF,NESTEQ,NW)
CALL MIGRHI_IMAGEZ (VBUF, BUF)
10 CONTINUE
CALL MIGRHI_IMAGEF (VBUF, BUF)
CALL APEX_APWR
RETURN
C
C MULTISTEP EXTRAPOLATION FOR JWSTAT = 1
C
ENTRY MIGRHI_MLTST1 (VBUF, BUF)
REWIND LDVVEL
CALL MIGRHI_IMAGEI
CALL MIGRHI_IMAGEA (VBUF, BUF)
DO 20 IZ = 1, NZ1
READ (LDVVEL) VBUF
CALL APEX_APPUT (VBUF, IVBUF, NX, 2)
CALL APEX_APWD
CALL MIGRHI_FPS_EXTPOL (NX,IAMBF,IAOLD,IQBUF,IWBUF,NESTEQ,NW)
CALL MIGRHI_IMAGEA (VBUF, BUF)
20 CONTINUE
CALL MIGRHI_IMAGAF (VBUF, BUF)
CALL APEX_APWR
RETURN
END

SUBROUTINE MIGRHI_IMAGEZ (XBUF, BUF)
C
C IMAGE THE WAVE-FIELD, AND REMOVE IT FROM THE QBUF.
C
INCLUDE 'MIGRHI.CMB/NOLIST'
C
DIMENSION XBUF(NX), BUF(NX, NTOUT)
C
11 DO 20 IZ = 1, NLOUT
JZOUT = JZOUT + 1
IF (JZOUT .GT. NTOUT) RETURN
CALL APEX_APWR
CALL MIGRHI_FPS_IMAGEW (NX, NW, IQBUF, IVBUF)
CALL APEX_APWR
CALL APEX_APGET (XBUF, IVBUF, NX, 2)
CALL APEX_APWD
CALL MIGRHI_FPS_PHSHT (NX, NW, ISBUF, IQBUF)
CALL MIGRHI_VVADD (NX, XBUF, BUF(1,JZOUT), BUF(1,JZOUT))
20 CONTINUE
GO TO 99
C
ENTRY MIGRHI_IMAGEF (XBUF, BUF)
IF (JZOUT .GE. NTOUT) RETURN
NLOUT = NTOUT - JZOUT
GO TO 11

```

```

ENTRY      MIGRHI_IMAGEI
JZOUT      = 0
NLOUT      = NZMINR
GO TO 99

C
C      IMAGE FOR JWSTAT = 1
C
ENTRY      MIGRHI_IMAGEA (XBUF, BUF)

22    DO 30 IZ = 1, NLOUT
      JZOUT = JZOUT + 1
      IF (JZOUT .GT. NTOUT) RETURN
      CALL APEX_APWR
      CALL MIGRHI_FPS_IMAGEW (NX, NW, IQBUF, IVBUF)
      CALL APEX_APWR
      CALL APEX_APGET (BUF(1,JZOUT), IVBUF, NX, 2)
      CALL APEX_APWD
      CALL MIGRHI_FPS_PSHFT (NX, NW, ISBUF, IQBUF)
30    CONTINUE
      GO TO 99

C
ENTRY      MIGRHI_IMAGAF (XBUF, BUF)
IF (JZOUT .GE. NTOUT) RETURN
NLOUT      = NTOUT - JZOUT
GO TO 22
99    RETURN
END

```

```

SUBROUTINE MIGRHI_TM2FRQ (BUF, TRACE, THDR)

C      TRANSFORM TO FREQUENCY DOMAIN AND SAVE ON QBUF IN MUX-ORDER

INCLUDE      'MONFORT/NOLIST'
INCLUDE      'MIGRHI.CMB/NOLIST'
INTEGER      THDR (THDRLEN)
LOGICAL      VIRGIN
DATA         VIRGIN / .TRUE. /

C
DIMENSION    BUF(2)

C
IF (VIRGIN)      THEN
      NW2      = NW + NW
      IBLSZ   = MIN0 (3*NW2, 32764)
      OPEN (UNIT = LDVHDR, FILE = FILHDR, TYPE = 'SCRATCH',
      FORM = 'UNFORMATTED')
      OPEN (UNIT = LDVFRQ, FILE = FILFRQ, TYPE = 'SCRATCH',
      FORM = 'UNFORMATTED', RECL = NW2, RECORDTYPE = 'FIXED',
      BLOCKSIZE = IBLSZ, INITIALSIZE = NX/2, BUFFERCOUNT = 2
      KYV1ST   = MAX0 (KYV1ST, THDR(KEYIND))
      KEYNXT  = KYV1ST
      NX       = 0
      NFOLDS  = MAXAPS / NTFET
      IFOLDS  = 0
      VIRGIN  = .FALSE.

ENDIF

```

```

C
KEYVAL = THDR (KEYIND)
IF (KEYVAL.LT.KEYNXT .OR. KEYVAL.GT.KYVLST)      RETURN
NZEROT = (KEYVAL - KEYNXT) / KYVINC

C
C CORRECT FOR ZERO TRACES
C

IF (NZEROT .GT. 0)      THEN
    DO 10 ITRACE = 1, NZEROT
        NX = NX + 1
        THDR (KEYIND) = KEYNXT
        KEYNXT = KEYNXT + KYVINC
        WRITE (LDVHDR) THDR
        IFOLDS = IFOLDS + 1
        IF (IFOLDS .GT. NFOLDS) THEN
            CALL MIGRHI_FTRANS (NFOLDS, BUF)
            IFOLDS = 1
        ENDIF
        CALL MIGRHI_STORE (NT, BUF((IFOLDS-1)*NT+1), 0.)
10    CONTINUE
    ENDIF

C
C NOW TREAT THE INPUT TRACE.
C

IFOLDS = IFOLDS + 1
IF (IFOLDS .GT. NFOLDS) THEN
    CALL MIGRHI_FTRANS (NFOLDS, BUF)
    IFOLDS = 1
ENDIF
CALL MIGRHI_MOVE (NT, TRACE, BUF((IFOLDS-1)*NT+1))
NX = NX + 1
THDR (KEYIND) = KEYNXT
WRITE (LDVHDR) THDR
KEYNXT = KEYNXT + KYVINC
RETURN

C
C END-PROCEDURE

C
ENTRY MIGRHI_TM2FRF(BUF)

C
CALL MIGRHI_FTRANS (IFOLDS, BUF)
RETURN
END

C
SUBROUTINE MIGRHI_FTRANS (NTRACE, BUF)
C
TRANSFORM NTRACE DATA TO FREQUENCY
C
INCLUDE      'MIGRHI.CMB/NOLIST'
C
DIMENSION BUF(2)
LOGICAL VIRGIN
DATA      VIRGIN / .TRUE. /
C
C ***** SET UP

```

```

C
IF (VIRGIN)      THEN
    NW2      = NW + NW
    IWO      = 2 * IFIX(SWO / DW + 0.5)
    VIRGIN   = .FALSE.
ENDIF
CALL APEX_APINIT (0, 0, ISTAT)
CALL APEX_APPUT (BUF, 0, NTRACE * NT, 2)
CALL APEX_APWD
CALL MIGRHI_FPS_RFFTMM (NTRACE, NT, NTFFT, NW2, IWO,
                         NTRACE*NT, NTRACE*NTFFT)
+ CALL APEX_APWR
CALL APEX_APGET (BUF, IWO, NTRACE*NW2, 2)
CALL APEX_APWD
CALL APEX_APRlse
C
C *** MOVE THE RESULT TO QBUF IN MUX-FORM
C
J1      = 1
DO 50 IX = 1, NTRACE
    J2      = J1 + NW2 - 1
    WRITE (LDVFRQ) (BUF(J), J = J1, J2)
50    J1      = J2 + 1
RETURN
END

SUBROUTINE MIGRHI_GETMXC (LDEV, CBUF, NX, NW)
C
C GET FREQUENCY DATA IN MUX-ORDER.
C
COMPLEX      CBUF(NX,NW)
C
REWIND LDEV
DO 10 IX = 1, NX
10    READ  (LDEV) (CBUF(IX,JW), JW = 1, NW)
RETURN
END

SUBROUTINE MIGRHI_OUTPUT (TRACE, THDR, IFLAG, BUF)
C
C PASS THE MIGRATED RESULT. ONE TRACE AT A TIME.
C
INCLUDE      'MONFORT/NOLIST'
INCLUDE      'MIGRHI.CMB/NOLIST'
DIMENSION    THDR(THDRLEN)
DIMENSION    BUF (NX, NTOUT)
LOGICAL      VIRGIN
DATA        VIRGIN / .TRUE. /
C
C
IF (VIRGIN)      THEN
    IX      = 1
    REWIND LDVHOR
    VIRGIN = .FALSE.

```

```

    ELSE
        IX      = IX + 1
    ENDIF
    IF (IX = NX) 10, 20, 30
10   IFLAG = FLG$MULTO
    GO TO 22
20   IFLAG = FLG$NORM
22   CALL MIGRHI_MOVEJ (NTOUT, BUF(IX,1), TRACE, NX, 1)
    READ (LDVHDR) THDR
    RETURN
30   IFLAG = FLG$MULTI
    RETURN
END

```

SUBROUTINE MIGRHI\_VELEDT (TIME, VRMS, VINT, XCNTRL, BUF)

C  
C  
C  
EDIT VELOCITY DATA.

```

INCLUDE      'MIGRHI.CMB/NOLIST'
DIMENSION    TIME(2), VRMS(2), VINT(2), XCNTRL(2), BUF(NZ1, 2)
C
REWIND LDVVEL
DDZ      = DZ * 1000.
DO 20 JCNTRL = 1, NCNTRL
READ (LDVVEL) XCNTRL(JCNTRL), NVAL, (TIME(I),VRMS(I),I=1,NVAL)
IF (IVTYPE .EQ. 0) THEN
    CALL MIGRHI_RMS2MV (NVAL, TIME, VRMS, VINT)
ELSE
    CALL MIGRHI_INT2MV (NVAL, TIME, VRMS, VINT)
ENDIF
I1      = 1
DO 10 I = 1, NVAL
I2      = MINO (NZ1, IFIX(TIME(I)/DDZ + 1.001))
IF (I1 .LE. I2) CALL MIGRHI_STORE (I2-I1+1, BUF(I1,JCNTRL),
+                                         VINT(I))
10   I1      = I2 + 1
    IF (I1 .LE. NZ1) CALL MIGRHI_STORE (NZ1-I1+1, BUF(I1, JCNTRL),
+                                         VINT(NVAL))
20   CONTINUE
CLOSE (LDVVEL)
OPEN (UNIT = LDVVEL, FILE = FILVEL, TYPE = 'SCRATCH',
+ FORM = 'UNFORMATTED', RECL = NX, RECORDTYPE = 'FIXED',
+ BLOCKSIZE = 8*NX, INITIALSIZE = NZ/2, BUFFERCOUNT = 2)
C
C     NOW INTERPOLATE IN THE HORIZONTAL DIRECTION.
C     NOW VRMS=XOUT AND TIME=VOUT.
C
CALL MIGRHI_VRAMP (NX, VRMS, FLOAT(KYV1ST), FLOAT(KYVRMSC))
DO 30 IZ = 1, NZ1
CALL MIGRHI_MOVEJ (NCNTRL, BUF(IZ, 1), VINT, NZ1, 1)
CALL MIGRHI_LINVBP(NCNTRL, XCNTRL, VINT, NX, VRMS, TIME)
30   WRITE (LDVVEL) (TIME(I), I = 1, NX)
RETURN
END

```

```

SUBROUTINE MIGRHI_RMS2MV (N, TIME, VRMS, VMIG)
C
C      TRANSFORM RMS VELOCITIES TO MIGRATION VELOCITIES, I.E.,
C      MIG VELOCITY = 1. / (0.5 * INTERVAL VELOCITY)
C
DIMENSION TIME(N), VRMS(N), VINT(N), VMIG(N)
VMIG(1) = 2.0 / VRMS(1)
IF (N .LT. 2) GO TO 99
DO 10 I = 2, N
HOLD = VRMS(I)**2 * TIME(I) - VRMS(I-1)**2 * TIME(I-1)
IF (HOLD .LE. 0.) STOP 'ERROR IN RMS VELOCITY'
10 VMIG(I) = 2. * SQRT((TIME(I) - TIME(I-1)) / HOLD)
GO TO 99
C
C      TRANSFORM INTERVAL VELOCITIES TO MIGRATION VELOCITIES.
C
ENTRY MIGRHI_INT2MV (N, TIME, VINT, VMIG)
C
DO 20 I = 1, N
20 VMIG(I) = 2.0 / VINT(I)
RETURN
END

SUBROUTINE MIGRHI_LINVBP (NINPT, XIN, YIN, NOUTPT, XOUT, YOUT)
C
C      LINEAR INTERPOLATION OF VARIABLE BASE POINTS.
C
DIMENSION XIN(NINPT), YIN(NINPT), XOUT(NOUTPT), YOUT(NOUTPT)
C
DO 20 J = 1, NOUTPT
XX      = XOUT(J)
IF (XX .LE. XIN(1))      THEN
    YOUT(J) = YIN(1)
ELSE
    DO 10 I = 2, NINPT
        IF (XX .LE. XIN(I))      GO TO 12
10        CONTINUE
        YOUT(J) = YIN(NINPT)
        GO TO 20
12        SLOPE = (YIN(I) - YIN(I-1)) / (XIN(I) - XIN(I-1))
        YOUT(J) = SLOPE * (XX - XIN(I)) + YIN(I)
ENDIF
20        CONTINUE
RETURN
END

;
;
;
;
; ***** * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
;
; SUBROUTINES WRITTEN IN VAX 11/780 ASSEMBLER LANGEAGE
;
; ***** * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
;
; .TITLE MIGRHI_MARSUB
;
```

```

.IDENT / 65.03 /
;

; .ENTRY MIGRHI_MOVE,^M<R8,R9,R10,R11>
MOVL @4(AP),R8 ; N
MOVL 8(AP),R9 ; A
MOVL 12(AP),R11 ; B
CLRL R10 ; SET UP INDEX
MOVE_LOOP:
MOVL (R9)[R10],(R11)[R10] ; MOVE A TO B
AOBLSS R8,R10,MOVE_LOOP
RET
;

; .ENTRY MIGRHI_MOVEJ,^M<R4,R5,R6,R7,R8,R9,R10,R11>
MOVL @4(AP),R11 ; N
MOVL 8(AP),R10 ; A
MOVL 12(AP),R9 ; B
MOVL @16(AP),R8 ; JMPA
MOVL @20(AP),R7 ; JMPB
CLRL R6 ; SET UP A-INDEX(JA)
CLRL R5 ; SET UP B-INDEX(JB)
CLRL R4 ; SET UP LOOP COUNT
MOVEJ_LOOP:
MOVL (R10)[R6],(R9)[R5] ; B(JB) = A(JA)
ADDL2 R8,R6 ; INCREMENT JA
ADDL2 R7,R5 ; INCREMENT JB
AOBLSS R11,R4,MOVEJ_LOOP
RET
;

; .ENTRY MIGRHI_STORE,^M<R8,R9,R10,R11>
MOVL @4(AP),R8 ; N
MOVL 8(AP),R9 ; X
MOVL @12(AP),R11 ; CONST
CLRL R10 ; SET UP X-INDEX
STORE_LOOP:
MOVL R11,(R9)[R10] ; STORE CONST TO X(I)
AOBLSS R8,R10,STORE_LOOP
RET
;

; .ENTRY MIGRHI_VINVRS,^M<R8,R9,R10,R11>
MOVL @4(AP),R8 ; N
MOVL 8(AP),R9 ; A
MOVL 12(AP),R11 ; B
CLRL R10 ; SET UP INDEX
VINVRS_LOOP:
DIVF3 (R9)[R10],#^F1.0,(R11)[R10] ; TAKE INVERS
AOBLSS R8,R10,VINVRS_LOOP
RET
;

; .ENTRY MIGRHI_VRAMP,^M<R7,R8,R9,R10,R11>
MOVL @4(AP),R11 ; N
MOVL 8(AP),R10 ; X
MOVL @12(AP),R9 ; X0

```

```

    MOVL    @16(AP),R8          ; DEL_X
    CLRL    R7                  ; SET UP INDEX
VRAMP_LOOP:
    MOVL    R9,(R10)[R7]
    ADDF2  R8,R9
    A0BLSS R11,R7,VRAMP_LOOP
    RET

;
;

    .ENTRY  MIGRHI_VSMULT,^M<R7,R8,R9,R10,R11>
    MOVL    @4(AP),R11          ; N
    MOVL    8(AP),R10           ; A
    MOVL    @12(AP),R9          ; S
    MOVL    16(AP),R8           ; C
    CLRL    R7                  ; CLEAR LOOP COUNT
VSMULT_LOOP:
    MULF3  R9,(R10)[R7],(R8)[R7]   ; C(I) = S * A(I)
    A0BLSS R11,R7,VSMULT_LOOP
    RET

;
;

    .ENTRY  MIGRHI_MOVEJC,^M<R4,R5,R6,R7,R8,R9,R10,R11>
    MOVL    @4(AP),R11          ; N
    MOVL    8(AP),R10           ; A
    MOVL    12(AP),R9           ; B
    MOVL    @16(AP),R8           ; JMPA
    MOVL    @20(AP),R7           ; JMPB
    CLRL    R6                  ; SET UP A-INDEX(JA)
    CLRL    R5                  ; SET UP B-INDEX(JB)
    CLRL    R4                  ; SET UP LOOP COUNT
MOVEJC_LOOP:
    MOVQ    (R10)[R6],(R9)[R5]     ; B(JB) = A(JA)
    ADDL2  R8,R6                ; INCREMENT JA
    ADDL2  R7,R5                ; INCREMENT JB
    A0BLSS R11,R4,MOVEJC_LOOP
    RET

;
;

    .ENTRY  MIGRHI_VVADD,^M<R7,R8,R9,R10,R11>
    MOVL    @4(AP),R11          ; N
    MOVL    8(AP),R10           ; A
    MOVL    12(AP),R9           ; B
    MOVL    16(AP),R8           ; C
    CLRL    R7                  ; SET UP LOOP COUNT
VVADD_LOOP:
    ADDF3  (R10)[R7],(R9)[R7],(R8)[R7]   ; C(I) = A(I) + B(I)
    A0BLSS R11,R7,VVADD_LOOP
    RET

;
;

    .END

```

```

"      SUBROUTINES WRITTEN IN AP-120B ASSEMBLER LANGUAGE
"
"      * * * * * * * * * * * * * * * * * * * * * * * * * * *
"
"      $TITLE    EXTPOL
"      $ENTRY    EXTPOL,7
"      $EXT     EXTPL1,EXPL2
"
"      S-PAD DEFINITION SECTION
"
NX      $EQU    0
IAMBF   $EQU    1
IAOLD   $EQU    2
IQBUF   $EQU    3
IWBUF   $EQU    4
NESTDF  $EQU    5
NW      $EQU    6
"
JQBUF   $EQU    IQBUF
JSW     $EQU    IWBUF
"
CTRNW   $EQU    6
CTRDFC  $EQU    7
EGTEEN  $EQU    8.
"
SP0     $EQU    0
SP1     $EQU    1
SP2     $EQU    2
SP3     $EQU    3
SP4     $EQU    4
SP5     $EQU    5
"
"      SAVE NW AND NESTDF TO DPX(4) AND DPX(-5)
"
EXTPOL: INCOPA
        MOV      NW,NW; DPX(3)<SPFN;
        DECOPA
        DECOPA
        MOV      NESTDF,NESTDF;DPX(-4)<SPFN;
        INCOPA
"
"
"      DO 20 I = 1, NW
"      CALL EXTPL1 (NX, IAMBF, *, **, JSW)
"
LOOP20: JSR     EXTPL1
"
"      DO 10 J = 1, NESTDF
"10     CALL EXTPL2 (NX, IAMBF, IAOLD, JQBUF, JSW, IIEXT(J))
"
        DECOPA
        INCOPA; DPY(-4)<DPX(-4)
        LDSPI   SP5; DB=14.                      "IIEXT(1) == 14.
"
LOOP10: JSR     EXTPL2
        LDSPI   EGTEEN; DB=18.
        ADD     EGTEEN,SP5; DECOPA
        LDSPI   CTRDFC; DB=DPY(-4)

```

```

DEC      CTRDFC; DPY(-4)<SPFN; INCOPA
BGT      LOOP10
"
"      RETURN TO LOOP 20
"
INC      JSW
ADD      NX,JQBUF
ADD      NX,JQBUF; INCOPA
LD SPI   CTRNW; DB=DPX(3)
DEC      CTRNW; DPX(3)<SPFN; DECDPA
BGT      LOOP20
RETURN
$END

```

```

$TITLE  EXTPL1
$ENTRY   EXTPL1
$EXT    VMUL,VSMUL
NX0     $EQU  0
IAMBF0  $EQU  1
IAOLDO  $EQU  2
JQBUFO  $EQU  3
JSW0    $EQU  4
"
IAMBF   $EQU  8.
IAOLD   $EQU  9.
JQBUF   $EQU  10.
JSW     $EQU  11.
"
SP0     $EQU  0
SP1     $EQU  1
SP2     $EQU  2
SP3     $EQU  3
SP4     $EQU  4
SP5     $EQU  5
SP6     $EQU  6
"
EXTPL1: MOV      IAMBF0,IAMBF
         MOV      IAOLDO,IAOLD
         MOV      JQBUFO,JQBUF
         MOV      JSW0,JSW
"
" CALL VSMUL (IVBUF, 1, JSW, IAMBF, 1, NX)
"
         MOV      NX0,SP5
         LD SPI  SP4;DB=1
         MOV      IAMBF,SP3
         MOV      JSW,SP2
         LD SPI  SP1;DB=1
         MOV      IAMBF,SP0
         SUB      SP5,SP0
         JSR      VSMUL
"
" CALL VMUL (IAMBF, 1, IAMBF, 1, ISMSBF, 1, NX)
"
         MOV      SP5,SP6

```

```

LDSP1    SP5;DB=1
MOV      IAMBF,SP4
ADD      SP6,SP4
LDSP1    SP3;DB=1
MOV      IAMBF,SP2
LDSP1    SP1;DB=1
MOV      IAMBF,SP0
JSR      VMUL
"
" RETURN INPUT VARIABLES
"
MOV      SP6,NX0
MOV      IAMBF,IAMBF0
MOV      IAOLD,IAOLDO
MOV      JQBUF,JQBUFO
MOV      JSW,JSWO
RETURN
$END

$TITLE  EXTPL2
$ENTRY   EXTPL2,6
$EXT    DIV,VSMUL,VMSA,CVAMMA
"
NX0     $EQU    0
IAMBF0  $EQU    1
IAOLDO  $EQU    2
JQBUFO  $EQU    3
JSWO    $EQU    4
JEXT0   $EQU    5
"
NX      $EQU    7.
IAMBF   $EQU    8.
IAMBF1  $EQU    IAMBF
IAMS81  $EQU    IAMBF
IAOLD   $EQU    9.
IAOLD2  $EQU    IAOLD
IAOLD3  $EQU    IAOLD
TWO     $EQU    10.
THREE   $EQU    TWO
FOUR    $EQU    TWO
CTRBN   $EQU    TWO
JQBUF   $EQU    11.
JEXT   $EQU    12.
JSW    $EQU    13.
NX2    $EQU    14.
"
SP0     $EQU    0
SP1     $EQU    1
SP2     $EQU    2
SP3     $EQU    3
SP4     $EQU    4
SP5     $EQU    5
SP6     $EQU    6
"
EXTPL2: MOV      NX0,NX
"
" IAMS8F = IAMBF + NX

```

```

MOV      IAMBF0,IAMBF
MOV      IAOLD0,IAOLD
MOV      JQBUF0,JQBUF
MOV      JSW0,JSW
MOV      JEXT0,JEXT
MOVL    NX,NX2

"
" SAVE S-PADS
"

"
" CALL CFBND2 (IAMBF, IANEW, IBNEW, JQBUF1, JQBUF, JSW, ADX)
"

MOV      IAMBF,SP0
MOV      IAOLD,SP1          "IANEW = IAOLD + NX2 + NX2
ADD     NX2,SP1
ADD     NX2,SP1
MOV      SP1,SP2          "IBNEW = IANEW + NX2
ADD     NX2,SP2
MOV      JQBUF,SP3          "JQBUF1 = JQBUF + 2
INC     SP3
INC     SP3
MOV      JQBUF,SP4
MOV      JSW,SP5
LD SPI   SP6;DB=16.        "ADX = 16 + JEXT
ADD     JEXT,SP6
JSR     CFBND2

"
LD SPI   CTRBN;DB=2        "SET UP CTR FOR CFBND2

"
BOUNDARY CODES FOR MIGRATION
"

SEE PAGE 43-R EQ. (11)

"

AMREAL $EQU 0
ACOF   $EQU 1
BCOF   $EQU 2
QINNER $EQU 3
QOUTER $EQU 4
SW     $EQU 5
ADX    $EQU 6

"
"

"DZ    == 1
"WIMAG == 2
"AMIMAG == 5
"EYEB == ADX + 1.

"
"

LOOPBN: LDMA;DB=5          "FETCH AMIMAG;
LDTMA;DB!=ONE               "FETCH 1.0
LDMA;DB=1                   "FETCH DZ
MOV SW,SW;SETMA;             "FETCH SW;
DPY<MD                      "SAVE AMIMAG
LDMA;DB=2                   "FETCH WIMAG
INC ADX;SETMA;              "FETCH EYEB;
DPX<MD                      "HOLD DZ

```

```

DEC ADX;SETMA;           "FETCH ADX;
                           FMUL DPX,MD          "CWDZ REAL
MOV AMREAL,AMREAL;SETMA;   "FETCH AMREAL;
                           FMUL DPX,MD          "CWDZ IMAG
DPX(3)<MD;DPY(3)<MD;      "SAVE EYEB;
                           FMUL;
                           FADD ZERO,MD        "DEN IMAG FILL
DPY(-4)<FM;              "SAVE CWDZ REAL;
DPX<MD;                  "HOLD ADX;
                           FMUL;
                           FADD FM,ZERO        "DEN REAL FILL
DPY(-3)<FM;              "SAVE CWDZ IMAG;
                           FMUL DPX,MD          "CAMX REAL;
                           FADD FM,FA           "DEN IMAG ADD
                           FMUL DPX,DPY         "CAMX IMAG
                           FMUL
BR .+2;                   "SAVE CAMX REAL;
DPX(-4)<FM;              "DEN REAL ADD
                           FMUL;
                           FADD FM,FA
LDRBN1: BR LOOPBN          "SAVE CAMX IMAG;
DPX(-3)<FM;              "DEN IMAG ADD
                           FADD FM,FA
DPY(-2)<FA;              "SAVE DEN REAL;
DPX(-2)<FA;              "SAVE CA+CW REAL;
                           FSUB DPY(-4),DPX(-4) "CW-CA REAL
DPY(-1)<FA;              "SAVE DEN IMAG;
                           FMUL DPY(-2),DPY(-2); "A*A;
                           FSUBR DPX(3),FA       "CA+CW IMAG
DPX(-4)<FA;              "SAVE CW-CA REAL;
                           FMUL DPY(-1),DPY(-1); "B*B;
                           FADD
                           FMUL;
                           FSUBR DPX(3),FA       "CA+CW-EB IMAG
DPY<FM;                  "SAVE CA+CW-EB IMAG;
                           FMUL;
                           FSUB DPY(-3),DPX(-3) "CW-CA IMAG
DPX(-1)<FA;              "SAVE CA+CW-EB IMAG;
                           FADD FM,DPY
DPY(0)<TM;                "1.0 AT NUMERATOR;
DPX(-3)<FA;              "SAVE CW-CA IMAG;
                           FADD
DPX(0)<FA                 "A*A + B*B AT DENOM
JSR DIV                   "CW-CA-EB IM
BR .+2;                   "DEN REAL
                           FMUL DPX(0),DPY(-2); "DEN REAL
                           FADD
LDRBN2: BR LDRBN1          "SAVE CW-CA-EB IM;
DPX(2)<FA;                "DEN IMAG
                           FMUL DPX(0),DPY(-1)
                           FMUL;
                           FADD DPX(-3),DPY(3)  "CW-CA+EB IM
DPY(-2)<FM;              "SAVE SCALE REAL;
                           FMUL FM,DPX(-2);     "XIN REAL;

```

```

        FADD
MOV QINNER,QINNER;SETMA;
    DPY(-1)<FM;
    DPX(-3)<FA;
        FMUL FM,DPX(-2);
        FADD DPY(3),FA
INC QINNER;SETMA;
    FMUL DPY(-1),DPX(-1);
        FADD
    FMUL DPY(-2),DPX(-1);
        FADD FM,ZERO
    DPY(-4)<MD;
    FMUL DPY(-2),DPX(-4);
        FSUBR FM,ZERO
    DPY(-3)<MD;
    FMUL DPY(-1),DPX(-4);
        FADD FM,FA
    FMUL DPY(-1),DPX(-3);
        FADD FM,FA
    DPX(-2)<FA;
    FMUL DPY(-2),DPX(-3);
        FADD FM,ZERO
    DPX(-1)<FA;
    FMUL DPY(-3),FA;
        FSUBR FM,ZERO
BR      .+2;
    FMUL DPY(-3),DPX(-2);
        FADD FM,FA
LDRBN3: BR      LDRBN2
    FMUL DPY(-4),DPX(-2);
        FADD FM,FA
MOV BCOF,BCOF;SETMA;MI<FA;
    FMUL DPY(-4),DPX(-1);
        FSUBR FM,ZERO
INC BCOF;SETMA;MI<FA;
    FMUL DPY(-2),DPX(-4);
        FADD FM,ZERO
    FMUL DPY(-2),DPX(2);
        FADD FM,FA
MOV QOUTER,QOUTER;SETMA;
    FMUL DPY(-1),DPX(2);
        FADD FM,FA
INC QOUTER;SETMA;
    DPX(-4)<FA;
    FMUL DPY(-1),DPX(-4);
        FSUBR FM,ZERO
    DPX(-3)<FA;
        FMUL;
        FSUBR FM,ZERO
    DPY(-2)<MD;
        FMUL;
        FSUBR FM,FA
    DPY(-1)<MD;
        FADD FM,FA
    DPX<FA;

```

"FETCH QINNER;  
 "SAVE SCALE IMAG;  
 "SAVE CW-CA+EB IM;  
 "XIN IMAG-;  
 "CA-CW+EB IM  
 "FETCH QINNER IMAG;  
 "XIN REAL;  
 "XIN IMAG+;  
 "XIN REAL FILL  
 "SAVE QIN REAL;  
 "XRT REAL;  
 "XIN IMAG FILL  
 "SAVE QIN IMAG;  
 "XRT IMAG-;  
 "XIN REAL ADD  
 "XRT REAL;  
 "XIN AIMG ADD  
 "SAVE XIN REAL;  
 "XRT IMAG+;  
 "XRT REAL FILL  
 "SAVE XIN IMAG;  
 "QINX REAL-;  
 "XRT IMAG FILL  
 "QINX IMAG;  
 "XRT REAL ADD  
 "QINX REAL+;  
 "XRT IMAG ADD  
 "WRITE BCOF REAL;  
 "QINX IMAG;  
 "QINX REAL FILL  
 "WRITE BCOF IMAG;  
 "XOUT REAL-;  
 "QINX IMAG FILL  
 "XOUT IMAG-;  
 "QINX REAL ADD  
 "FETCH QOUTER REAL;  
 "XOUT REAL-;  
 "QINX IMAG ADD  
 "FETCH QOUTER IMAG;  
 "SAVE QINX REAL;  
 "XOUT IMAG+;  
 "XOUT REAL FILL  
 "SAVE QINX IMAG;  
 "XOUT IMAG FILL  
 "SAVE QOUTER REAL;  
 "XOUT REAL SUBR  
 "SAVE QOUTER IMAG;  
 "XOUT IMAG ADD  
 "HOLD XOUT REAL;

```

        FMUL DPY(-2),FA;           "QBNX REAL+;
                                FADD DPX(-4),ZERO      "ACOF REAL FILL
        DPX(1)<FA;              "HOLD XOUT IMAG;
        FMUL DPY(-2),FA;           "QBNX IMAG;
                                FADD DPX(-3),ZERO      "ACOF IMAG FILL
        BR    .+2;                FMUL DPY(-1),DPX(1)       "QBNX REAL-
LDRBN4: BR    LDRBN3          FMUL DPY(-1),DPX;         "QBNX IMAG;
                                FADD FM,FA            "ACOF REAL ADD
        FMUL;                  FADD FM,FA            "ACOF IMAG ADD
        FMUL;                  FSUBR FM,FA          "ACOF REAL SUBR
                                FADD FM,FA          "ACOF IMAG ADD
        MOV ACOF,ACOF;SETMA;MI<FA;      "WRITE ACOF REAL;
                                FADD
        INC ACOF;SETMA;MI<FA           "WRITE ACOF IMAG
" CALL CFBND2 (IAMB9, IANEW9, IBNEW9, JQBUF8, JQBUF9, JSW, IDZ)
"
        MOVL    NX,NX2             "RECREATE NX2
        BR    .+2;    MOV SP5,JSW      "RECREATE JSW
LDRBN5: BR    LDRBN4          "IAMB9 = IAMB + NX - 1
        ADD    NX,SP0
        DEC    SP0
        MOV    NX2,SP6             "MAKE SP6 = NX2-3
        DEC    SP6
        DEC    SP6
        DEC    SP6
        ADD    SP6,SP1             "IANEW9 = (IANEW+1) + (NX2-3)
        ADD    SP6,SP2             "IBNEW9 = (IBNEW+1) + (NX2-3)
        ADD    SP6,SP4             "JQBUF9 = (JQBUF+1) + (NX2-3)
        MOV    SP4,SP3             "JQBUF8 = JQBUF9 - 2
        DEC    SP3
        DEC    SP3
        "
        "      "SP5"               "INVARIANT
        LDSP1  SP5;DB=16.          "ADX == 16 + JEXT
        ADD    JEXT,SP6
        JSR    CFBND2
"
        DEC    CTRBN
        BGT    LDRBN5
" CALL VSMUL (JEXT(3), 4, JSW, IAIW, 1, 4) SCR = (0, 3, 15)
"
        MOV    JEXT,SP0             "JEXT(3) = JEXT + 2
        INC    SP0
        INC    SP0
        LDSP1  SP1;    DB=4
        MOV    JSW,SP2
        LDSP1  SP3;    DB=10.        "IAIW == 10.
        LDSP1  SP4;    DB=1
        LDSP1  SP5;    DB=4
        JSR    VSMUL
" CALL VSMSA (IAMS1, 1, JEXT(2), JEXT, IAOLD2, 2, NY)
"

```

```

ADD      NX,IAMBF          "IAMS81 = IAMBF + 1
INC      IAMBF
MOV      IAMS81,SP0
LDSP1    SP1;   DB=1
MOV      JEXT,SP2          "JEXT(2) = JEXT + 1
INC      SP2
MOV      JEXT,SP3
LDSP1    SP5;   DB=2
ADD      SP5,IAOLD          "IAOLD2 = IAOLD + 2
MOV      IAOLD2,SP4
MOV      NX,SP6              "NY = NX - 2
SUB      SP5,SP6
JSR      VMSA

"
" CALL VMSA (IAMS81, 1, JEXT(6), JEXT(5), IANEW2, 2, NY)
"

MOV      IAMS81,SP0
"      "SP1"    INVARIANT
LDSP1    FOUR;DB=4
ADD      FOUR,SP2
ADD      FOUR,SP3
MOV      IAOLD2,SP4          "IANEW2=IAOLD2+NX2*2
ADD      NX2,SP4
ADD      NX2,SP4
"      "SP5,SP6"    INVARIANT
JSR      VMSA

"
" CALL VMSA (IAMS81, 1, JEXT(10), JEXT(9), IBOLD2, 2, NY)
"

MOV      IAMS81,SP0
"      "SP1"    "INVARIANT"
ADD      FOUR,SP2
ADD      FOUR,SP3
MOV      IAOLD2,SP4          "IBOLD2 = IAOLD2 + NX2
ADD      NX2,SP4
"      "SP5,SP6"    "INVARIANT"
JSR      VMSA

"
" CALL VMSA (IAMS81, 1, JEXT(14), JEXT(13), IBNEW2, 2, NY)
"

MOV      IAMS81,SP0
"      "SP1"    "INVARIANT"
ADD      FOUR,SP2
ADD      FOUR,SP3
MOV      IAOLD2,SP4          "IBNEW2=IAOLD2+NX2*3
ADD      NX2,SP4
ADD      NX2,SP4
ADD      NX2,SP4
"      "SP5,SP6"    "INVARIANT
JSR      VMSA

"
" CALL VMSA (IAMBF1, 1, JEXT(4), IAIW, IAOLD3, 2, NY)
"

SUB      NX,IAMS81          "IAMBF1 = IAMS81 - NX
MOV      IAMBF1,SP0

```

```

"      "SP1"    "INVARIANT
LDSP1  SP3;    DB=10.          "IAIW == 10.
SUB    SP3,SP2
INC    IAOLD2
MOV    IAOLD3,SP4
"      "SP5,SP6"    "INVARIANT
JSR    VSMSA
CALL   VSMSA (IAMBF1, 1, JEXT(8), IAIW1, IANEW3, 2, NY)
"
MOV    IAMBF1,SP0
"      "SP1"    "INVARIANT
ADD    FOUR,SP2
INC    SP3
MOV    IAOLD3,SP4
ADD    NX2,SP4
ADD    NX2,SP4
"      "SP5,SP6"    "INVARIANT
JSR    VSMSA
"
CALL   VSMSA (IAMBF1, 1, JEXT(12), IAIW2, IBOLD3, 2, NY)
"
MOV    IAMBF1,SP0
"      "SP1"    "INVARIANT
ADD    FOUR,SP2
INC    SP3
MOV    IAOLD3,SP4
ADD    NX2,SP4
"      "SP5,SP6"    "INVARIANT
JSR    VSMSA
"
CALL   VSMSA (IAMBF1, 1, JEXT(16), ISIW3, IBNEW3, 2, NY)
"
MOV    IAMBF1,SP0
"      "SP1"    "INVARIANT
ADD    FOUR,SP2
INC    SP3
MOV    IAOLD3,SP4
ADD    NX2,SP4
ADD    NX2,SP4
ADD    NX2,SP4
"      "SP5,SP6"    "INVARIANT
JSR    VSMSA
"
-- RESET IAOLD3 TO IAOLD
"
LDSP1  THREE;DB=3
SUB    THREE,IAOLD3
"
CALL   CVAMMA (NX, JQBUF, IAOLD, IBOLD, IAOLD)
"
MOV    NX,SP0
MOV    JQBUF,SP1
MOV    IAOLD,SP2
MOV    IAOLD,SP3
ADD    NX2,SP3          "IBOLD = IAOLD + NX2

```

```

        MOV      IAOLD,SP4
        JSR      CVAMMA
"
" SOLVE TRIDIAGONAL SIMULTANEOUS EQUATIONS
"
" CALL TRIDGX (NX, JQBUF, IANEW, IBNEW, IAOLD)
"
        MOV      NX,SP0
        MOV      JQBUF,SP1
        MOV      IAOLD,SP2                                "IANEW = IAOLD+NX2*2
        ADD      NX2,SP2
        ADD      NX2,SP2
        MOV      SP2,SP3                                "IBNEW = IANEW + NX2
        ADD      NX2,SP3
        MOV      IAOLD,SP4
        MOV      JSW,SP6                                "SAVE JSW IN SP6
"
" SUBROUTINE TRIDGX (N, T, A, B, D)
" COMPLEX   T(N), A(N), B(N), D(N), DEN
" DO 10 I = 2, N-1
" DEN     = 1. / (B(I) + A(I) * B(I-1))
" B(I)    = -A(I) * DEN
"10      A(I)    = (D(I) - A(I) * A(I-1)) * DEN
" T(N)    = (A(N-1) * B(N) + A(N)) / (1. - B(N) * B(N-1))
" DO 20 J = 1, N-1
" I       = N - J
"20      T(I)    = B(I) * T(I+1) + A(I)
" RETURN
"END
"
N      $EQU    0
T      $EQU    1
A      $EQU    2
B      $EQU    3
D      $EQU    4
CTR   $EQU    5
"
" DATA PAD VARIABLES * * * * * * * * * * * * * *
"
" DPX(-4,-3) : A(I) PRESTORED
" DPY(-4,-3) : B(I) PRESTORED
" DPX(-2,-1) : D(I) PRESTORED
"           : T(I+1) PRESTORED
" DPY(-2,-1) : DEN
" DPX( 0, 1) : TEMPORARY REGISTERS/ JSR DIV
" DPY( 0, 1) : TEMPORARY REGISTERS/ JSR DIV/ A(I)
" DPX( 2, 3) : A(I-1) PRESTORED
" DPY( 2, 3) : B(I-1) PRESTORED
"
" * * * * * * * * * * * * * * * * * * * *
"
        MOV B,B; SETMA                                "FETCH B(1) RL
        INC B ; SETMA                               "FETCH B(1) IM
        INC B ; SETMA                               "FETCH B(2) RL
        INC B ; SETMA;                            "FETCH B(2) IM

```

```

        DPY(2)<MD          " STORE B(1) RL
MOV A,A; SETMA;      "FETCH A(1) RL
        DPY(3)<MD          " STORE B(1) IM
INC A ; SETMA;       "FETCH A(1) IM
        DPY(-4)<MD          " STORE B(2) RL
INC A ; SETMA;       "FETCH A(2) RL
        DPY(-3)<MD          " STORE B(2) IM
INC A ; SETMA;       "FETCH A(2) IM
        DPX(2)<MD          " STORE A(1) RL
CLR TWO;DPX(3)<MD    " STORE A(1) IM
INCL TWO;             "TWO = 2
        DPX(-4)<MD;        " STORE A(2) RL
                FMUL DPY(2),MD  "A2B1 REAL+
ADD TWO,D; SETMA;     "FETCH D(2) RL
        DPX(-3)<MD;        " STORE A(2) IM
                FMUL DPY(2),MD  "A2B1 IMAG
INC D ; SETMA;        "FETCH D(2) IM,
                FMUL DPY(3),DPX(-3)  "A2B1 REAL-
MOV N,CTR;             "A2B1 IMAG
                FMUL DPY(3),DPX(-4); "DEN REAL FILL
FADD FM,ZERO          "SET UP COUNTER
SUB TWO,CTR;           " STORE D(2) RL,
        DPX(-2)<MD;        "DEN IMG* FILL
                FMUL;          "STORE D(2) IM,
                FSUBR FM,ZERO   "DEN REAL ADD
ADD N,T;               "SET UP T(N),
        DPX(-1)<MD;        "DEN IMG* ADD
                FMUL;          "SAVE DEN REAL;
                FSUBR FM,FA     "SAVE DEN IMG*;
ADD N,T;               FMUL DPY(-4),FA   "DEN REAL ADD
                FSUBR DPY(-3),FA  "DEN IMG* ADD
                FADD             "SAVE DEN REAL;
                FADD DPY(-2),DPY(-2) "A*A
                FMUL DPY(-1),DPY(-1) "B*B
                FMUL             "PUSH
DPX<FM;               FADD FM,DPX   "TEMPORALILY HOLD,
                FMUL             "A*A + B*B
DPY(0)<TM;              FADD             "JSR DIV(NUM)=1.0
                DPX(0)<FA          "JSR DIV(DEN)=(AA+BB)
JSR DIV                 FMUL DPX,DPY(-2) "SCALE REAL
                FMUL DPX,DPY(-1) "SCALE IMAG
BR .+2;                  DPY(0)<DPX(-4); "SKIP LADDER
                                "MOVE A(I)R TO DPY(0)

```

```

        FMUL
LADDR2: BR      LOOP10
              DPY(-2)<FM;
                  FMUL FM,DPX(-4)
              DPY(-1)<FM;
                  FMUL FM,DPX(-4)
INC B; SETMA;
              DPY(1)<DPX(-3);
                  FMUL DPX(-3),DPY(-1)
INC B; SETMA;
              FMUL DPX(-3),DPY(-2);
                  FSUBR FM,ZERO
SUB TWO,B;
              FMUL DPY(0),DPX(2);
                  FSUBR FM,ZERO
              DPY(-4)<MD;
                  FMUL DPY(0),DPX(3);
                      FADD FM,FA
              DPY(-3)<MD;
                  FMUL DPY(1),DPX(3);
                      FSUBR FM,FA
DEC B; SETMA;MI<FA;
              DPY(2)<FA;
                  FMUL DPY(1),DPX(2);
                      FSUBR FM,ZERO
INC B; SETMA;MI<FA;
              DPY(3)<FA;
                  FMUL;
                      FSUBR FM,ZERO
BR      .+2;
                  FMUL;
                      FADD FM,FA
LADDR1: BR      LADDR2
ADD TWO,B;
              FSUBR FM,FA
              FADD DPX(-2),FA
              FADD DPX(-1),FA
INC A;SETMA;
              DPX<FA;
                  FMUL DPY(-2),FA;
                      FADD
INC A;SETMA;
              FMUL DPY(-2),FA
INC D;SETMA;
              FMUL DPY(-1),FA
INC D;SETMA;
              DPX(-4)<MD;
                  FMUL DPY(-1),DPX;
                      FADD FM,ZERO
SUB TWO,A;
              DPX(-3)<MD;
                  FMUL DPX(-4),DPY(2);
                      FADD FM,ZERO
              DPX(-2)<MD;
                  FMUL DPX(-4),DPY(3);
"SAVE SCALE REAL;
    "B(I) REAL-
"SAVE SCALE IMAG;
    "B(I) IMAG-
"FETCH NEXT B(I) RL,
    "MOV A(I) TO DPY(1);
    "B(I) REAL+
"FETCH NEXT B(I) IM,
    "B(I) IMAG-;
    "B(I) REAL FILL
"BACK TO B(I),
    "A1A2 REAL+;
    "B(I) IMAG FILL
"STORE B(I+1) RL,
    "A1A2 IMAG;
    "B(I) REAL ADD
"STORE B(I+1) IM,
    "A1A2 REAL-;
    "B(I) IMAG ADD
"WRITE B(I) RL,
    "FOR NEXT B(I-1) RL,
    "A1A2 IMAG;
    "A1A2 REAL FILL
"WRITE B(I) IM,
    "FOR NEXT B(I-1) IM,
                    "A1A2 IMAG FILL
"SKIP THE LADDER,
                    "A1A2 REAL ADD
"GO B(I+1),
    "A1A2 IMAG ADD
    "ADEN REAL ADD
    "ADEN IMAG ADD
"FETCH A(I+1) RL,
    "TEMPORARY,
    "CA REAL+;
"FETCH A(I+1) IM,
    "CA IMAG
"FETCH D(I+1) RL,
    "CA REAL-
"FETCH D(I+1) IM,
    "STORE A(I+1) RL,
    "CA IMAG;
    "CA REAL FILL
"BACK TO A(I),
    "STORE A(I+1) IM,
    "A3B2 REAL+;
    "CA IMAG FILL
"STORE D(I) RL,
    "A3B2 IMAG;

```

```

        FSUBR FM,FA          "CA REAL ADD
DPX(-1)<MD;
        FMUL DPX(-3),DPY(3);
        FADD FM,FA
DEC A;SETMA;MI<FA;
        DPX(2)<FA;
        FMUL DPX(-3),DPY(2);
        FADD FM,ZERO
INC A;SETMA;MI<FA;
        DPX(3)<FA;
        FMUL;
        FSUBR FM,ZERO      "DEN REAL FILL
DEC CTR;
        FMUL;               "DEN IMG* FILL-
ADD TWO,A;BGT LADDR1;
        FSUBR FM,FA          "DE REAL ADD
" END OF DO LOOP 10
"
" STATEMENT 15.
"
LDTMA;DB=!ONE
        DPX(-2)<DPY(-4)
        DPX(-1)<DPY(-3)
        FMUL DPX(-2),DPY(2)
        FMUL DPX(-2),DPY(3);
        FADD TM,ZERO
        FMUL DPX(-1),DPY(3);
        FADD
        FMUL DPX(-1),DPY(2);
        FSUBR FM,FA
        FMUL;               "IM,
        FADD FM,ZERO         "1. - RL
        FMUL;               "FILL IM
        FADD FM,FA          "ADD RL
        FADD FM,FA          "ADD IM
DPY(-2)<FA;
        FADD
DPY(-1)<FA;
        FMUL DPY(-2),DPY(-2)
        FMUL DPY(-1),DPY(-1)
        FMUL
DPX<FM;
        FMUL
        FADD FM,DPX         "TEMPORARY SAVE
DPY(0)<TM;
        FADD
DPX(0)<FA
JSR     DIV
        FMUL DPX,DPY(-2)
        FMUL DPX,DPY(-1)
" DENOMINATOR COMPLETED
"
" THE NUMERATOR CODES ARE:
"

```

```

        FMUL DPX(2),DPY(-3)
DPY(-2)<FM;
        FMUL DPX(2),DPY(-4)
DPY(-1)<FM;
        FMUL DPX(3),DPY(-4)
        FMUL DPX(3),DPY(-3);
        FADD FM,ZERO
FMUL;
        FADD FM,ZERO
FMUL;
        FADD FM,FA
FSUBR FM,FA
FADD DPX(-3),FA
FADD DPX(-4),FA
DPX<FA;
        FMUL DPY(-2),FA;
        FADD
        FMUL DPY(-2),FA
        FMUL DPY(-1),FA
MOV N,CTR;
        FMUL DPY(-1),DPX;
        FADD FM,ZERO
DEC CTR;
        FMUL;
        FADD FM,ZERO
SUB TWO,B;
        DPY(-3)<DPY(3);
        FMUL;
        FADD FM,FA
DEC B;
        DPY(-4)<DPY(2);
        FSUBR FM,FA
DEC T;SETMA;MI<FA;
        DPX(-1)<FA;
        FADD
DEC T;SETMA;MI<FA;
        DPX(-2)<FA
" END OF STATEMENT 15.
"
" DO LOOP 20
"
DEC A;
LOOP20: DEC A;SETMA;
        FMUL DPX(-1),DPY(-4)
        FMUL DPX(-2),DPY(-4)
DEC A;SETMA;
        FMUL DPX(-2),DPY(-3)
        FMUL DPX(-1),DPY(-3);
        FADD FM,ZERO
DEC B;SETMA;
        FMUL;
        FADD FM,ZERO
DEC B;SETMA;
        DPX(-3)<MD;
        FMUL;
        "ADJUST A-INDEX
        "IM
        "RL +
"FETCH A(I) IM,
        "IM
"FETCH A(I) RL,
        "RL -
        "FILL IM
"FETCH B(I-1) IM,
        "FILL RL
"FETCH B(I-1) RL,
        "STORE A(I) IM,

```

```

        FADD FM,FA          "ADD IM
        DPX(-4)<MD;
        FSUBR FM,FA         "STORE A(I) RL,
        DPY(-3)<MD;         "ADD RL
        FADD DPX(-3),FA     "STORE B(I-1) IM,
        DPY(-4)<MD;         "ADD IM
        FADD DPX(-4),FA     "STORE B(I-1) RL,
        DEC T;SETMA;MI<FA;   "ADD RL
        DPX(-1)<FA;         "WRITE T(I) IM,
        FADD                 "SAVE T(I-1) IM,
        DEC T;SETMA;MI<FA;   "WRITE T(I) RL,
        DPX(-2)<FA           "SAVE T(I-1) RL
        DEC CTR;
        BGT LOOP20;          FMUL DPX(-1),DPY(-4)      "IM
        FMUL DPX(-2),DPY(-4) "SEE IF DONE?????
                                         "RL +
" END OF DO LOOP 20.
"
" RECREATE INPUT VARIABLES
"
        MOV     NX,NX0
        DEC     IAMBF1
        MOV     IAMBF,IAMBF0
        MOV     IAOLD,IAOLDO
        MOV     JQBUF,JQBUFO
        MOV     SP6,JSW0
        MOV     JEXT,JEXT0
        RETURN
$END

"
$TITLE CVAMMA
$ENTRY CVAMMA
"
" D(I) = A(I) * (Q(I-1) + Q(I+1)) + B(I) * Q(I), I = 2, N
"
N $EQU 0
Q $EQU 1
A $EQU 2
B $EQU 3
D $EQU 4
CTR $EQU 0
TWO $EQU 5
"
" DP REGISTER MAP * * * * * * * * * * *
"
" CPX(-4,-3) : Q(I-1), Q(I)
" DPY(-4,-3) : Q(I), Q(I+1)
" DPX(-2,-1) : Q(I+1)
" DPY( 2, 3) : A(I)
"
CVAMMA: MOV Q,Q;SETMA          "FETCH Q(1) RL
        INC Q;SETMA          "FETCH Q(1) IM
        INC Q;SETMA          "FETCH Q(2) RL
        INC Q;SETMA          "FETCH Q(2) IM,

```

```

INC Q;SETMA;           FADD ZERO,MD          "FILL QSUM RL
INC Q;SETMA;           FADD ZERO,MD          "FETCH Q(3) RL,
                           DPX(-4)<MD          "FILL QSUM IM
INC A;                 DPX(-3)<MD          "FETCH Q(3) IM,
                           DPX(-2)<MD          "SAVE Q(2) RL
INC A;SETMA;           DPX(-1)<MD          "SKIP A(1),
                           FA                  "SAVE Q(2) IM
                           FADD DPX(-2),FA      "FETCH A(2) RL,
                           FA                  "SAVE Q(3) RL
                           FADD DPX(-1),FA      "FETCH A(2) IM,
                           FA                  "SAVE Q(3) IM,
                           FA                  "QSUM RL
INC D;                 FA                  "SKIP D(1),
                           FA                  "QSUM IM
DEC CTR;               DPY(2)<MD          "CTR = N-1,
                           FA                  "SAVE A(2) RL
DEC CTR;               DPY(3)<MD          "CTR = N-2,
                           FA                  "SAVE A(2) IM
INC B                 FA                  "SKIP B(1)

"
LOOP:                DPX<FA;
                           FMUL DPY(2),FA;      "HOLD QSUM RL,
                           FADD                 "A(I)*QSUM RP,
                           FA                  "PUSH QSUM IM
INC B;SETMA;           DPY(-4)<DPX(-2);      "FETCH B(I) RL,
                           FA                  "MOVE Q(I+1) TO Q(I)
                           FMUL DPY(2),FA      "A(I)*QSUM IM
INC B;SETMA;           DPY(-3)<DPX(-1);      "FETCH B(I) IM,
                           FA                  "MOVE Q(I+1) TO Q(I)
                           FMUL DPY(3),FA      "A(I)*QSUM RM
                           FMUL DPY(3),DPX;     "A(I)*QSUM IM,
                           FADD FM,ZERO        "FILL AQSUM RL
INC Q;SETMA;           DPY<MD;              "FETCH Q(I+2) RL,
                           FA                  "HOLD B(I) RL,
                           FMUL DPX(-4),MD;     "Q(I)*B(I) RP,
                           FADD FM,ZERO        "FILL AQSUM IM
INC Q;SETMA;           FA                  "FETCH Q(I+2) IM,
                           FMUL DPX(-4),MD;     "Q(I)*B(I) IM,
                           FSUBR FM,FA         "ADD AQSUM RL
INC A;SETMA;           FA                  "FETCH A(I+1) RL,
                           FMUL DPX(-3),MD;     "Q(I)*B(I) RM,
                           FADD FM,FA          "ADD AQSUM IM
INC A;SETMA;           DPX(-2)<MD;          "FETCH A(I+1) IM,
                           FA                  "SAVE Q(I+2) RL,
                           FMUL DPX(-3),DPY;    "Q(I)*B(I) IM,
                           FADD FM,FA          "ADD AQSQB RL
                           DPX(-1)<MD;          "SAVE Q(I+2) IM
                           FMUL;                "PUSH QB RL,
                           FADD FM,FA          "ADD AQSQB IM
                           DPY(2)<MD;           "SAVE A(I+1) RL,
                           FMUL;                "PUSH QB IM,
                           FSUBR FM,FA         "ADD AQSQB RM
                           DPY(3)<MD;           "SAVE A(I+1) IM,
                           FMUL;                "ADD AQSQB IM
INC D;SETMA;MI<FA;     FADD FM,FA          "WRITE D(I) RL,
                           FA                  "FILL QSUM RP
                           FADD DPX(-4),ZERO

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```

INC D;SETMA;MI<FA;           "WRITE D(I) IM,
                             FADD DPX(-3),ZERO   "FILL QSUM IM
DEC CTR;                      "CTR = CTR - 1,
DPX(-4)<DPY(-4);           "MOVE Q2 TO Q1 RL,
                             FADD DPX(-2),FA    "ADD QSUM RL
BGT LOOP;                     "SEE IF DONE????,
DPX(-3)<DPY(-3);           "MOVE Q2 TO Q1 IM,
                             FADD DPX(-1),FA    "ADD QSUM IM

RETURN
$END

$TITLE  CVCSML
$ENTRY   CVCSML,4
N        $EQU    0
X        $EQU    1
S        $EQU    2
Z        $EQU    3
CVCSML: MOV S,S;SETMA        "GET SR
MOV X,X;SETMA        "GET X1R
INC S;SETMA        "GET SI
INC X;SETMA;          "GET X1I;
DPY<MD;              "SAVE SR
DPX<MD;              "HOLD X1R;
                      "X1R*SR
FMUL DPY,MD
INC X;SETMA;          "GET X2R;
DPY(1)<MD;            "SAVE SI;
                      "X1R*SI
INC X;SETMA;          "GET X2I;
DPX(1)<MD;            "HOLD X1I;
                      "X1I*SI
FMUL DPY(1),MD
FMUL DPX(1),DPY;
FADD FM,ZERO          "X1I*SR;
                      "FILL Z1 REAL
                      "ADJUST Z-INDEX;
                      "HOLD X2R;

DEC Z;                  "FILL Z1 IMAG
DPX<MD;              "GET X3R;
                      "HOLD X2I;
                      "X2R*SR;
                      "Z1 REAL ADD

"
"
LOOP:  INC X;SETMA;          "GET X(I+2) IMAG;
FMUL DPX, DPY(1);      "X(I+1)R * SI;
FADD FM,FA              "Z(I) IMAG ADD
INC Z;SETMA;MI<FA;       "PUT Z(I) REAL;
FMUL DPX(1),DPY(1);     "X(I+1)I * SI;
FADD
INC Z;SETMA;MI<FA;       "PUT Z(I) IMAG;
DPX<MD;                 "HOLD X(I+2) REAL;
FMUL DPX(1),DPY;         "X(I+1)I * SR;
FADD FM,ZERO             "Z(I+1) R-FILL
DEC N;                  "AJUST LOOP COUNT;

```

```

DPX(1)<MD;
      FMUL;           FADD FM,ZERO          "Z(I+1) I-FILL
INC X;SETMA;           FMUL DPX,DPY;        "GET X(I+3) REAL;
                      FSUBR FM,FA;       "X(I+2)R * SR;
BGT LOOP             "Z(I+1) R-ADD
RETURN
$END

" " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " "
" " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " "
" " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " "
SUBROUTINE RFFTMM (NX, NT, NFFT, NW2, IWO, NXNT, NXNTFF)
" " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " "
" " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " "
$TITLE IMAGEW
$ENTRY IMAGEW,4
$EXT SVE

" " DO 10 IX = 1, NX
"10 CALL SVE (IQ+IX+IX-2, NX2, IG+IX-1, NW)
"
" "
NXIN $EQU 0
NWIN $EQU 1
QIN $EQU 2
GIN $EQU 3
"
NX $EQU 15.
NW $EQU 14.
Q $EQU 13.
TWO $EQU 12.
"
SP0 $EQU 0
SP1 $EQU 1
SP2 $EQU 2
SP3 $EQU 3
"
IMAGEW: MOV NXIN,NX          "SAVE NX
        MOV NWIN,NW          "SAVE NW
        MOV QIN,Q            "SAVE Q
        MOV GIN,SP2          "SAVE G
        MOV NX,SP1
        ADD NX,SP1
        LDSPI TWO; DB=2      "TWO = 2
        MOV Q,SP0
LOOP:  MOV NW,SP3
        JSR SVE
        ADD TWO,Q
        INC SP2
        DEC NX
        MOV Q,SP0; BGT LOOP
        RETURN
$END

```

```

$TITLE PHSHT
$ENTRY PHSHT,4
$EXT CVCML
"
" DO 10 J = 1, NW
"10 CALL CVCML (NX, IQ+NX2*(J-1), IS+J+J-2, IQ+NX2*(J-1))
"
NXIN $EQU 0
NWIN $EQU 1
SIN $EQU 2
QIN $EQU 3
"
SP0 $EQU 0
SP1 $EQU 1
SP2 $EQU 2
SP3 $EQU 3
"
NX $EQU 14.
NW $EQU 13.
Q $EQU 12.
NX2 $EQU 11.
"
PHSHFT: MOV NXIN,NX           "SAVE NX
          MOV NWIN,NW           "SAVE NW
          MOV QIN,Q              "SAVE Q
          MOVL NX,NX2            "NX2 = NX + NX
          MOV NX,SP0              "SET UP SP0
LOOP:   MOV Q,SP1              "SET SP1
          MOV Q,SP3
          JSR CVCML
          ADD NX2,Q
          INC SP2                "SP2 INCREMENTED 1 BY CVCML.
          DEC NW
          MOV NX,SP0; BGT LOOP
          RETURN
          SEND

```

```

$TITLE RFFTMM
$ENTRY RFFTMM,7
$EXT VCLR           "SCRATCH = SP0, SP15
$EXT VMOV           "SCRATCH = SP0, SP2, SP15
$EXT RFFT           "SCRATCH = SP2 TO SP15
"
" INPUT ARGUMENT DESCRIPTION.
"
INX $EQU 0
INT $EQU 1
INTFFT $EQU 2
INW2 $EQU 3
IIWO $EQU 4
INXNT $EQU 5
INXNTF $EQU 6
"
" INTERNAL SCRATCH VARIABLES DESCRIPTION.
"
```



```

LOOP10: BEQ EXIT10;
        MOV IMDDST,SP0
        MOV NCLEAR,SP2
JSR      VCLR
        SUB NCLEAR,IMODST
        MOV IMDSRC,SP0
        MOV IMDDST,SP2
JSR      VMOV
        SUB NT,IMDSRC
        SUB NT,IMDDST
        DEC CTR;
BR       LOOP10
EXIT10: MOV IMDDST,SP0
        MOV NCLEAR,SP2
JSR      VCLR
"
"
"
"     IMDSRC = 0
"     DO 20 J = 1, NX
"     CALL RFFT (IMDSRC, NTFFT, 1)
"20     IMDSRC = IMDSRC + NTFFT
"
"
"
"     DECDPA;                                "GO DPA(-1)
"     CLR SP0
"     INCDPA;                                "GO DPA;
"     MOV NX,CTR;                            "SET UP COUNTER;
"                                         "SAVE COUNTER
"     DPX(-4)<SPFN
"     MOV NTFFT,SP1
"     LDSPI SP2; DB=1
"     JSR RFFT
"     DECDPA;                                ADD SP1,SP0
"                                         LDSPI CTR; DB=DPX(-4)
"     INCDPA;                                DEC CTR; DPX(-4)<SPFN
"     BGT  LOOP20
"
"
"
"     IMDSRC = IWO
"     IMDDST = IWO
"     DO 30 J = 2, NX
"     IMDSRC = IMDSRC + NTFFT
"     IMDDST = IMDDST + NW2
"30     CALL VMOV (IMDSRC, 1, IMDDST, 1, NW2)
"
"
"
"     GET IMDSRC(=IWO), CTR(=NX), NTFFT, AND SP4(=NW2)
"
"
"
"     INCDPA; MOV SP1,NTFFT

```

```
LD SPI IMOSRC; DB=DPX(3)
INC DPA;
LD SPI CTR;      DB=DPY(3)
DEC DPA;
LD SPI SP4;      DB=DPY(3)
"
"
"
ADJUST COUNTER, RESET DPA, SET SP1, SP2 TO 1, 1.
"
"
"
DEC DPA;
DEC CTR
LD SPI SP1; DB=1
LD SPI SP3; DB=1
MOV IMOSRC,IMDOST
"
"
"
DO 30 J = 2, NX
"
"
"
LOOP30: BEG DONE;
ADD NTFFT,IMOSRC
ADD SP4, IMDOST
MOV IMOSRC,SP0
MOV IMDOST,SP2
JSR      VMOV
BR       LOOP30; DEC CTR
DONE:   RETURN
$END
```